Power Laws in the Atmosphere

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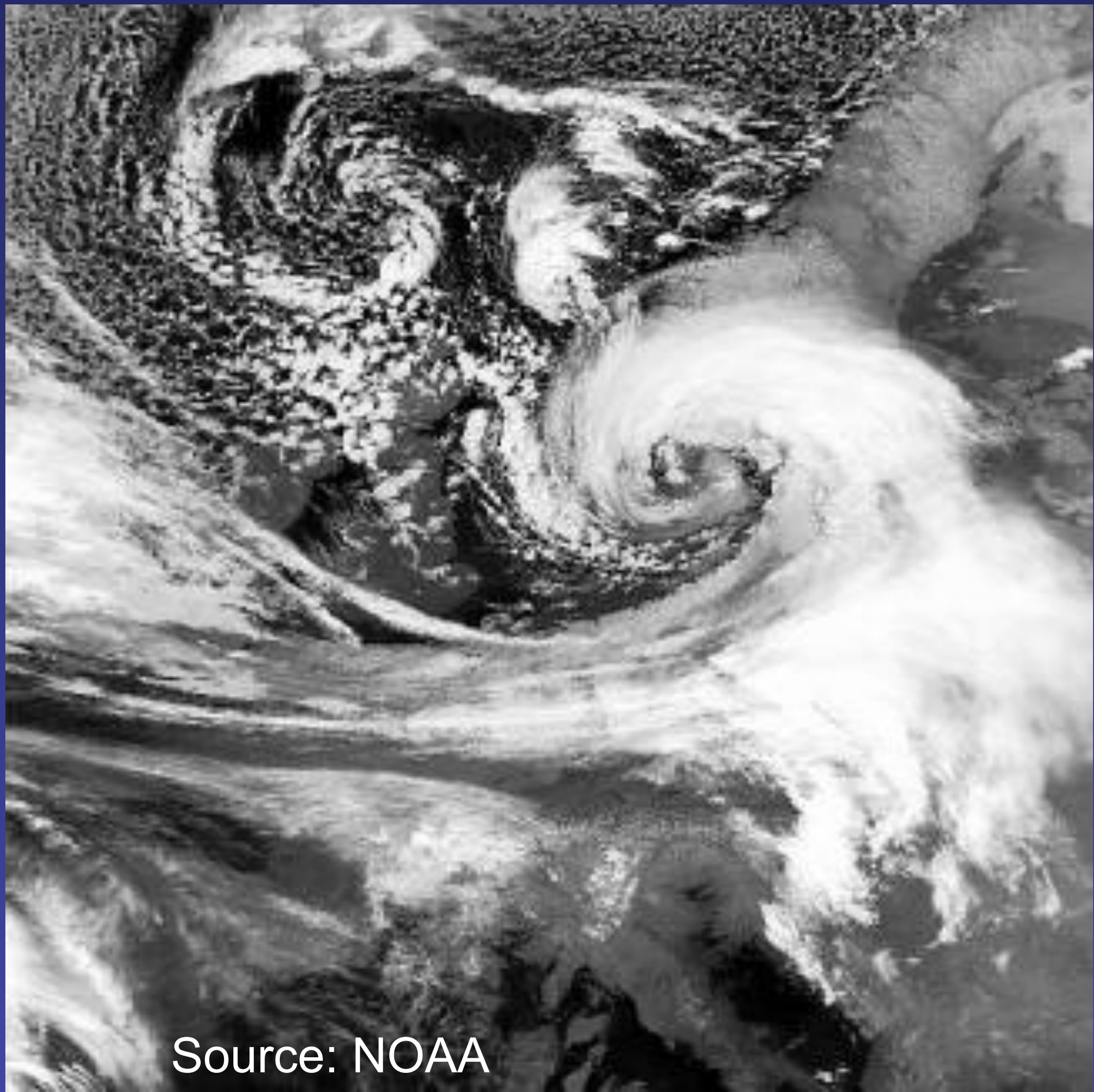
Motivation: Atmospheric complex phenomena

Synoptic scale



Meso-scale

Convective scale

6th ECSS, Palma (Mallorca),
3.-7. Oct. 2011

Former Work

Topic:

Investigation of frequency distributions of
low pressure systems on different scales

Central question:

How to find an appropriate intensity parameter?

First idea:

Parameter definition based on the horizontal
equation of motion

Introduction of an energy of displacement
(Schielicke and Névir, 2009)

How to describe vortices with help of the horizontal equation of motion?

Definition of a mass-specific energy of displacement E:

Radial path integral at the moment of maximum intensity describes the mass-specific work that was necessary for the generation of the system

$$E := \int_0^R a(r)dr = -\int_{P_0}^{P} \rho^{-1} dp$$

with $a(r) := (v(r))^2/r + fv(r)$

Explicit expression depends on the scale

Low Pressure System	Prevailing Balance	Energy of Displacement
Tornadoes	Cyclostrophic	E=v²/2
Tropical Cyclones	Gradient wind	E=v²+fvR
Extratropical Cyclones	geostrophic	E=fvR

Frequency distributions of vortices concerning their energy of displacement expression (Schielicke and Névir, 2009)

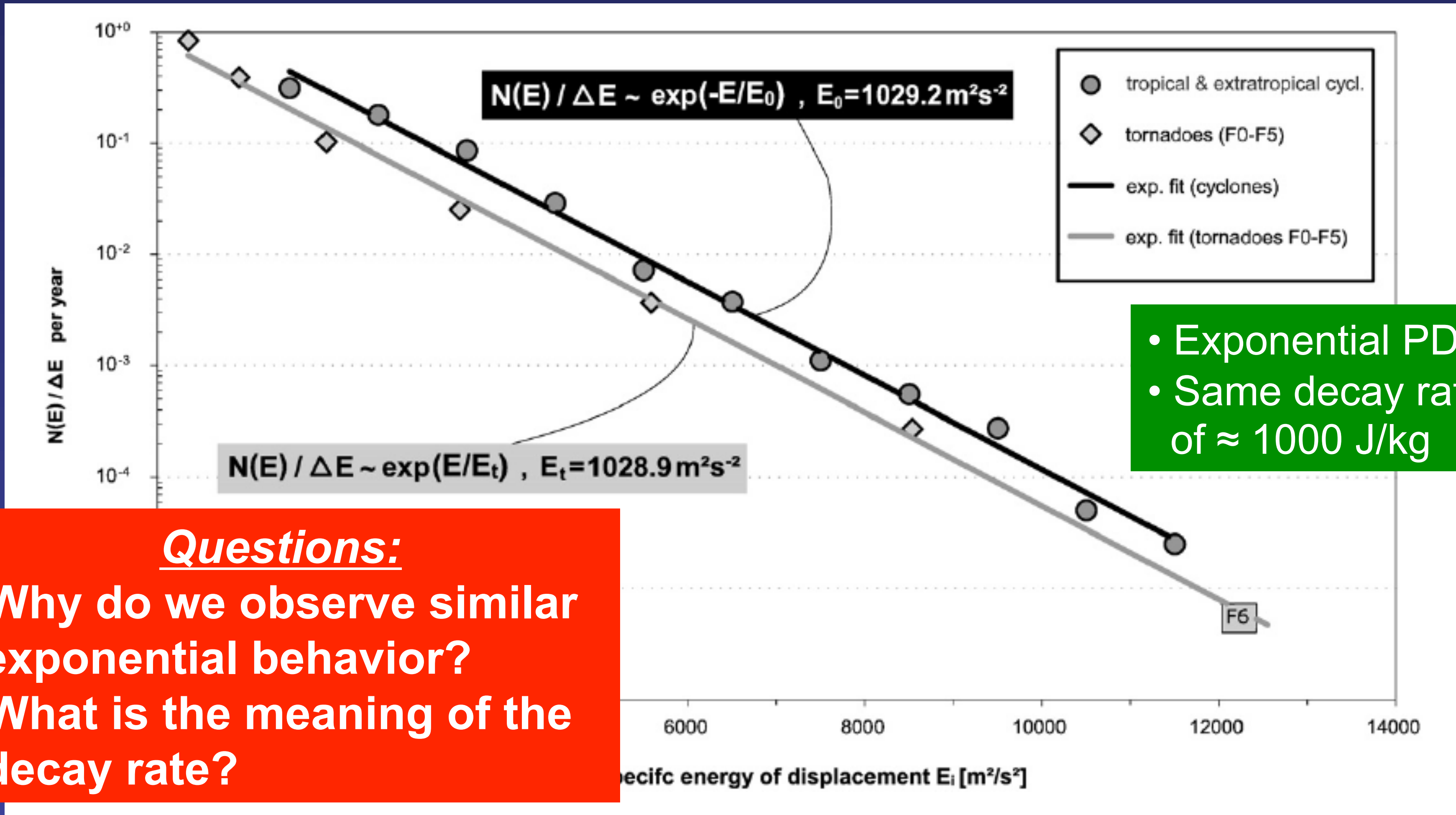


Fig. 4. Summary of density–intensity (energy of displacement) distributions per year of tornado data (1950–1999) and cyclone data (1958–1997), containing the summation of hurricanes and extra-tropical depressions (see Table 3). The distributions show the same exponential behavior, represented by the same characteristic, universal energy of displacement scale of $E_u \approx 1000\ m^2s^{-2}$ over the whole range.

- Exponential PDFs
- Same decay rate of ≈ 1000 J/kg

<u>Questions:</u>

- **Why do we observe similar exponential behavior?**
- **What is the meaning of the decay rate?**

Schielicke and Névir, 2009: On the theory of intensity distributions of tornadoes and other low pressure systems. *Atm. Res.*, **93**, 11-20.

Former and current Work

Central question:

Why can‘t we find power laws for
atmospheric features
although other criteria of complex
systems match for the atmosphere?

Established example: Gutenberg-Richter law of earthquakes

Is it possible to apply the seismic
moment concept to the atmosphere?

Introduction of an atmospheric moment

(Schielicke and Névir, 2011)

Seismic moment concept combines intensity (stress drop) with geometric properties.

↓

Application to atmospheric vortices:

Combination of the local intensity (the energy of displacement) with the mass affected during the life-cycle:

$$M_a := \overline{\rho} V E = -\overline{\rho} V \int_{P_0}^{P} \rho^{-1} dp$$

↓

Interpretation

Proportional to the total, mass-related work of generation.	Proportional to a heat amount due to external forcing?	Consideration of whole life-cycle; process-related

Application of moment concept to tornadoes

- Atmospheric moment M_a as product of total during life time affected mass M and energy of displacement E:

$$\boxed{M_a = ME}$$

$$\boxed{M = \overline{\rho}V = \overline{\rho}AL / C^{hv}} \qquad \boxed{E = v_F^2 / 2}$$

- Explicit expression for tornadoes:

$$\boxed{M_a \sim AL\overline{\rho}\left\langle v_F^2 \right\rangle / 2}$$

A: Area at max. intensity
L: Path length
ρ: density
v: velocity
C^{hv}: constant relating hor. and vertical extent

Frequency distributions of tornadoes and earthquakes concerning their moments (Schielicke and Névir, 2011)

Finally power laws!

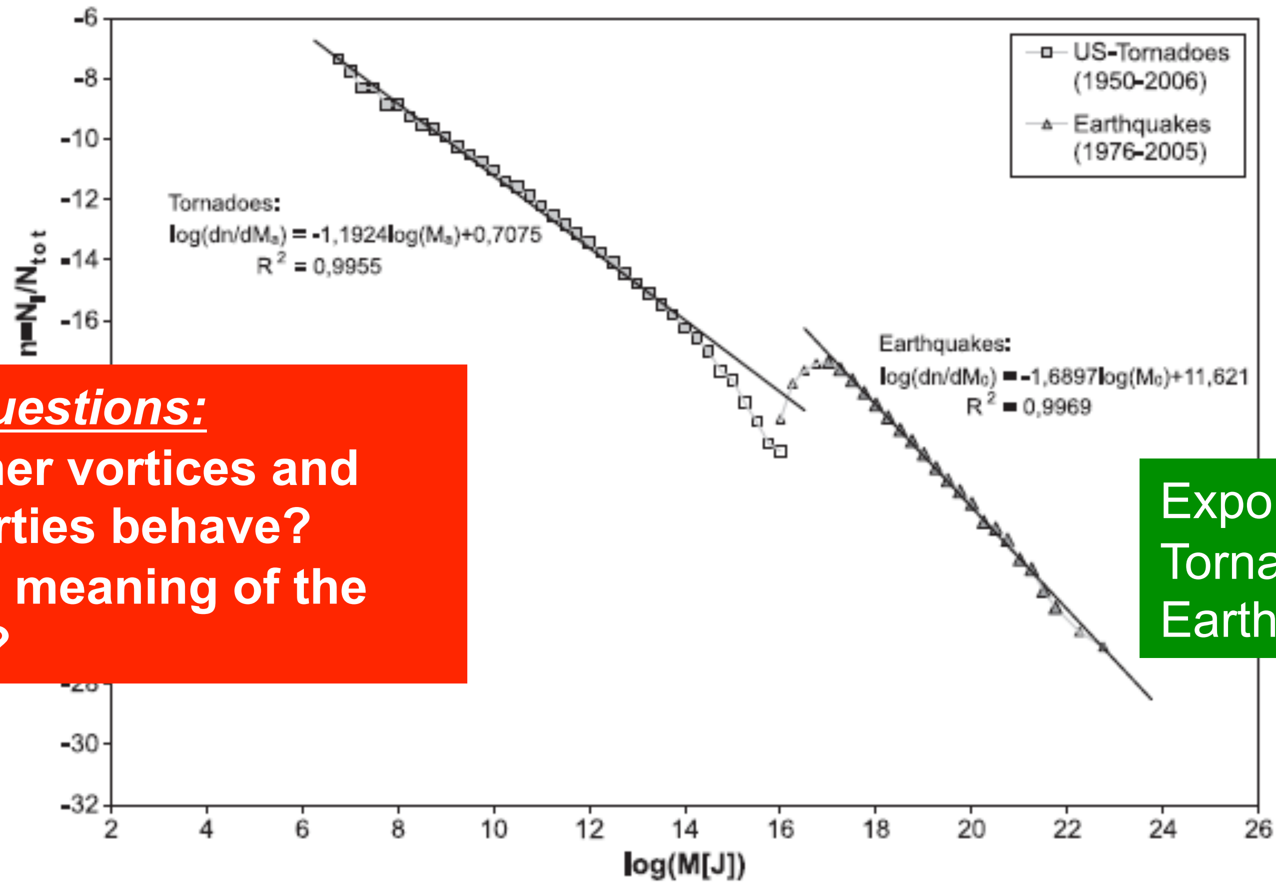


Fig. 5. Comparison of probability density distributions of tornadoes (1950–2006) and earthquakes (1976–2005) concerning their moments in a log–log plot. Linear fits have been applied to the data. The fitting ranges are indicated by the filled symbols.

Questions:
- **How do other vortices and their properties behave?**
- **What is the meaning of the exponents?**

Exponents for
Tornadoes: 1.2
Earthquakes: 1.7

Schielicke and Névir, 2011: Introduction of an atmospheric moment combining Eulerian and Lagrangian aspects of vortices. *Atm. Res.*, **100**, 357-365.

Application of atmospheric moment concept to synoptic-scale cyclones

- Local intensity (energy of displacement) can be estimated by $E \approx \rho^{-1} \Delta P$
- ΔP: Pressure deficit between vortex center and environment

$$M_a := \overline{\rho} V E = -\overline{\rho} V \int_{P_0}^{P} \rho^{-1} dp$$

- Estimation of total volume $V = L * 2R * H \sim L \pi R^2 = LA$
- $\overline{\rho} \approx$ const.

- Explicit expression of atmospheric moment for synoptic-scale vortices:

$$M_{a,cyc} \sim LA\Delta P$$

A: Area at maximum intensity
L: Path length
ΔP: Pressure deficit at maximum intensity
$\rho^{-1}\Delta P = - g_0 \Delta h$, $g_0 = 9.80665 m/s^2$

ERA-Interim data

- Spatial resolution of about 1.5°
- Temporal resolution: Δt=6h
- Time period: Jan-Dec 1989,
- 1000 hPa geopotential height field,
- 80°N-80°S

Method of cyclone tracking and area estimation

- *Identification* of cyclones as local minima in the geopotential height field
- *Tracking*: Nearest-neighbor search method (similar to Blender et al., 1997)
- *Area estimation*: outermost closed isobar (with help of contourplot algorithm of gnuplot)

Only a first guess!

Probability density distributions of lows – first results

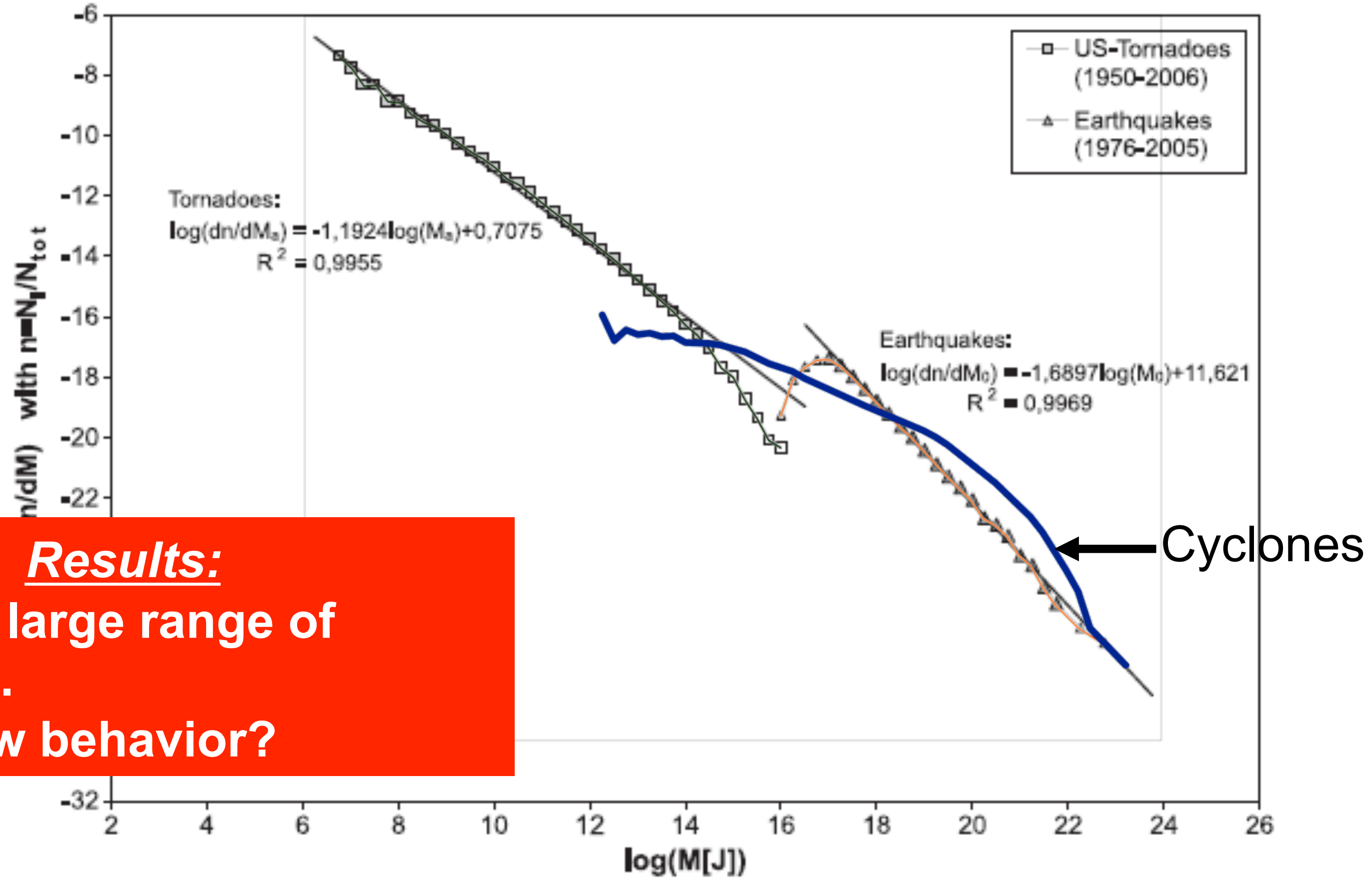
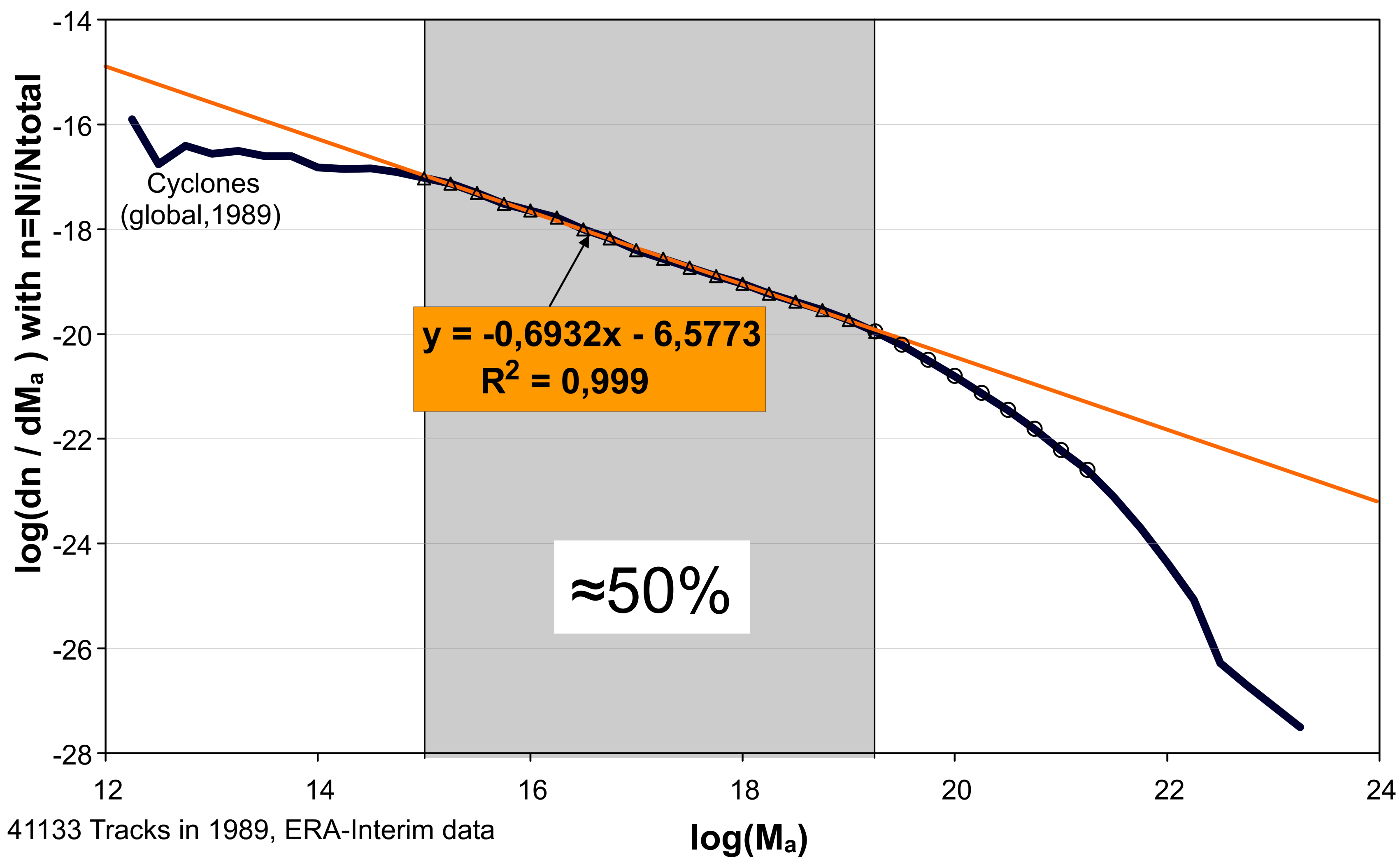


Fig. 5. Comparison of probability density distributions of tornadoes (1950–2006) and earthquakes (1976–2005) concerning their moments in a log–log plot. Linear fits have been applied to the data. The fitting ranges are indicated by the filled symbols.

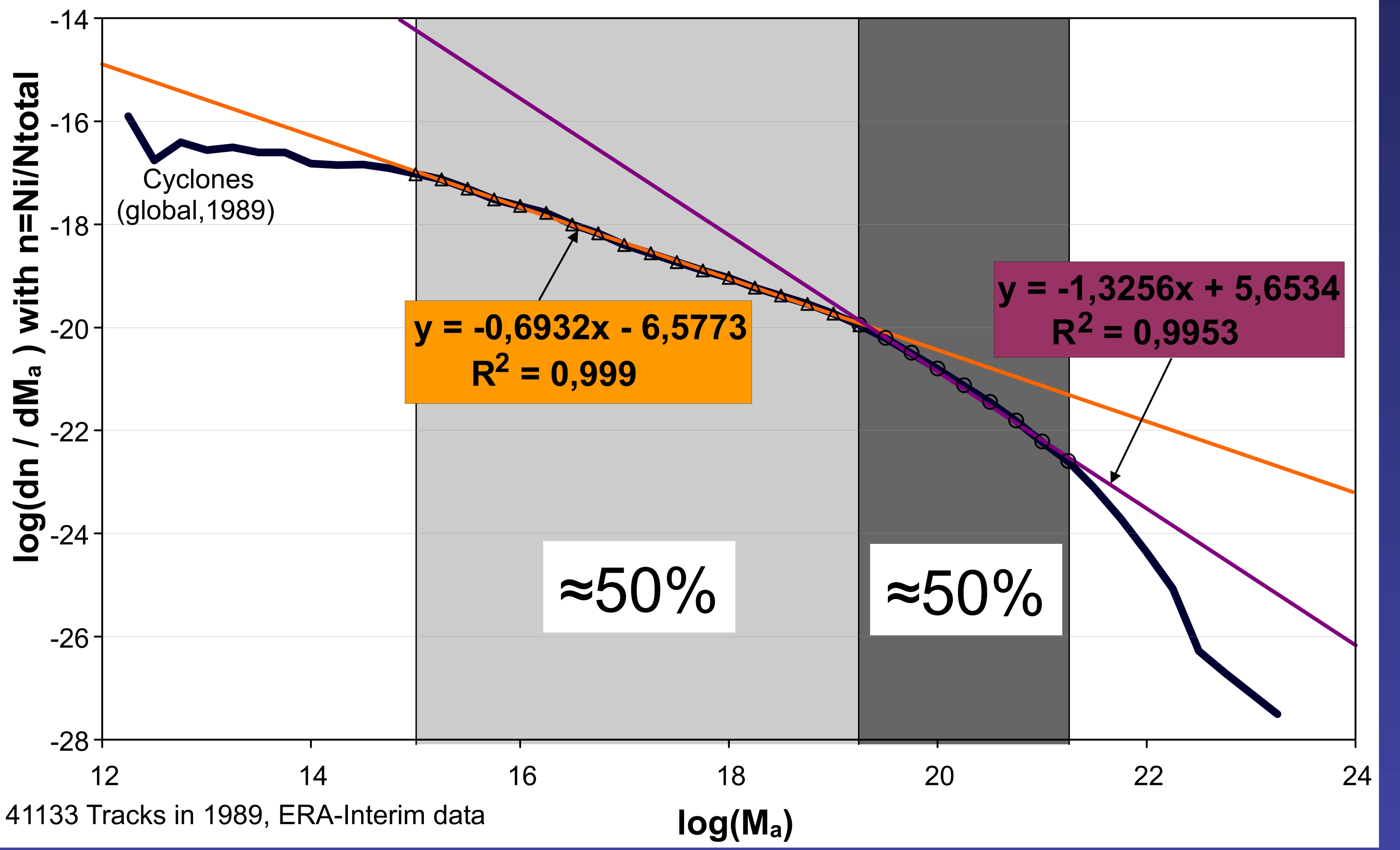
Results:

- **Covers a large range of moments.**
- **Power law behavior?**

Probability density distributions of lows – first results

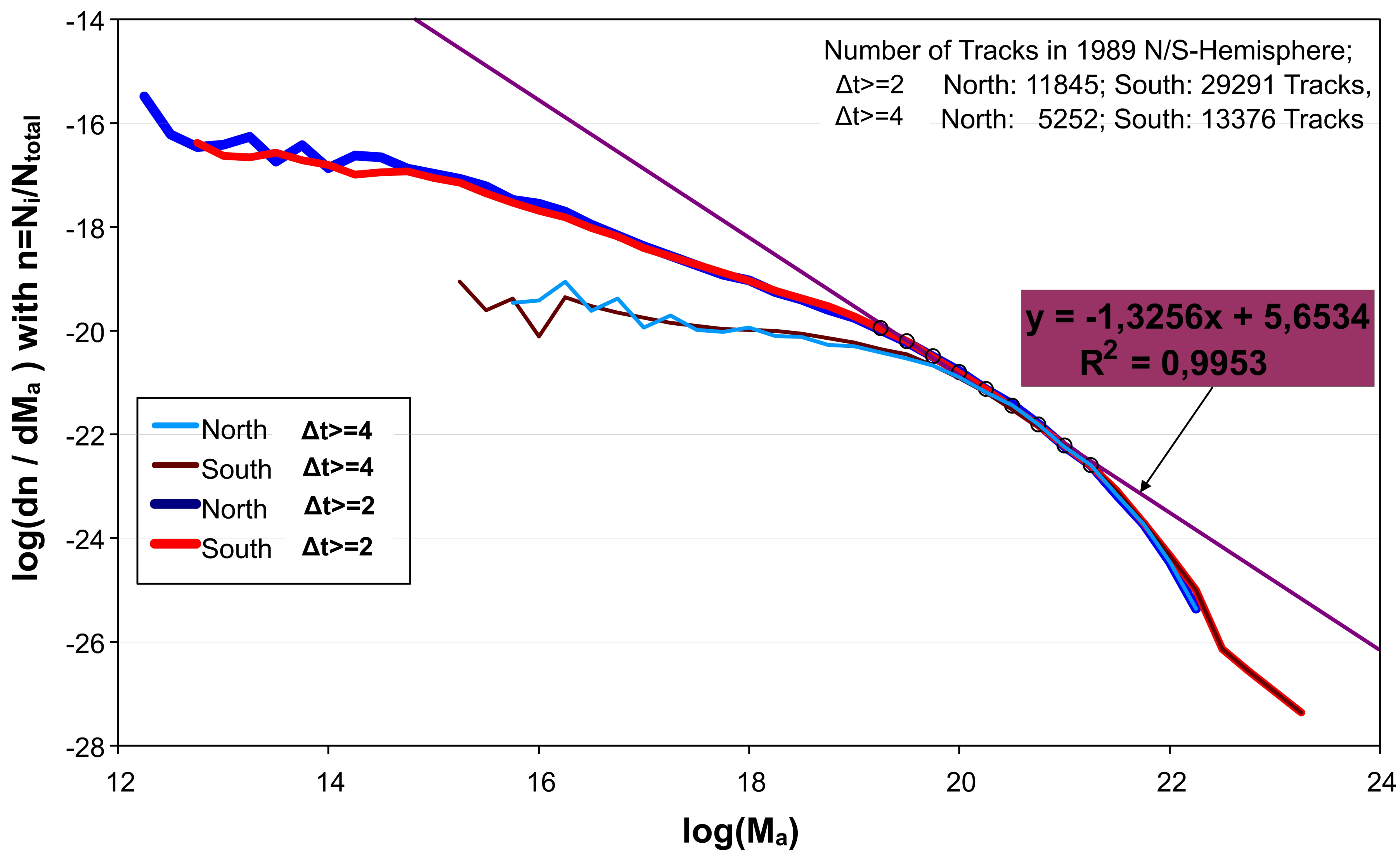


Probability density distributions of lows – first results

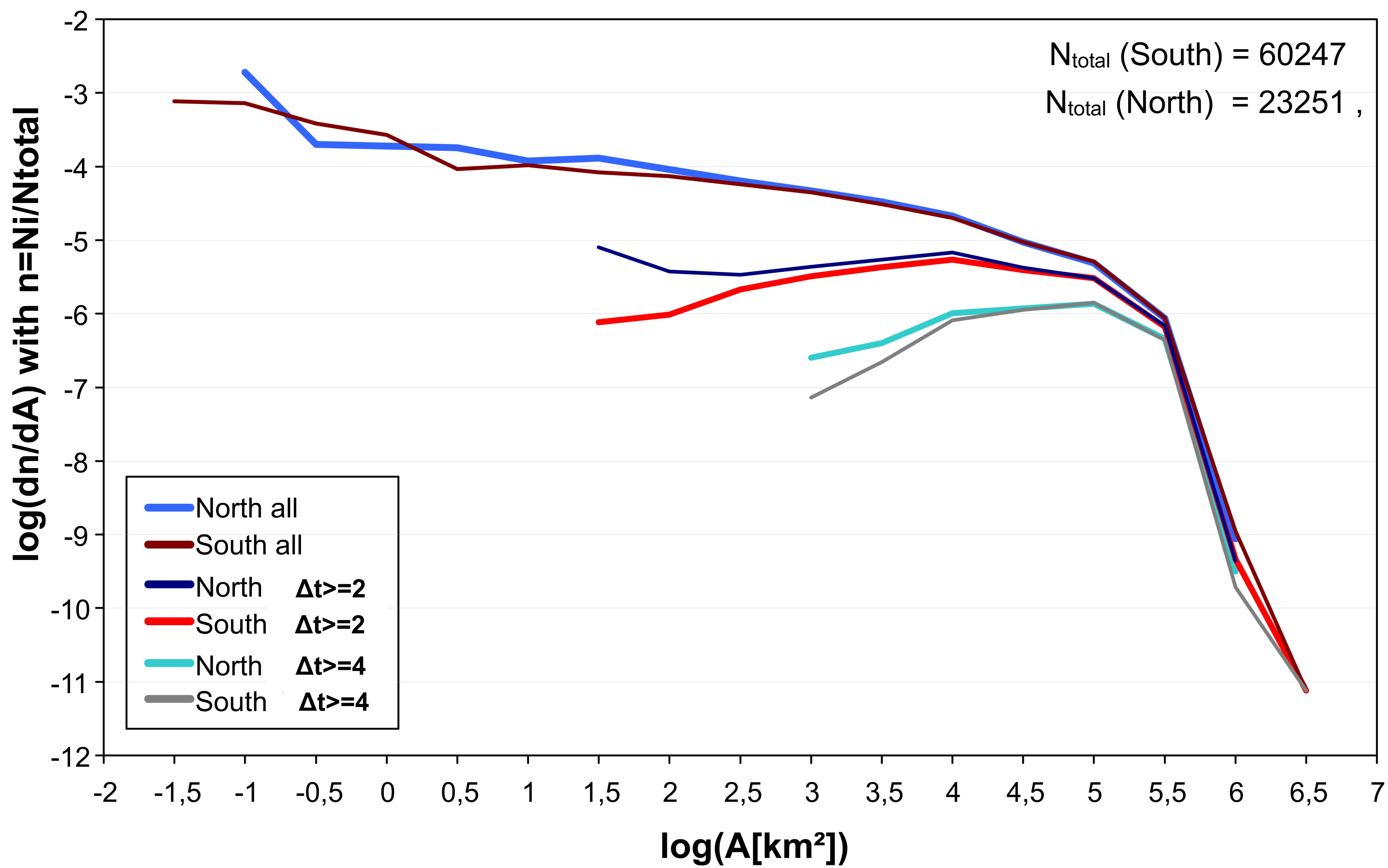


41133 Tracks in 1989, ERA-Interim data

Probability density distributions of cyclones – first results

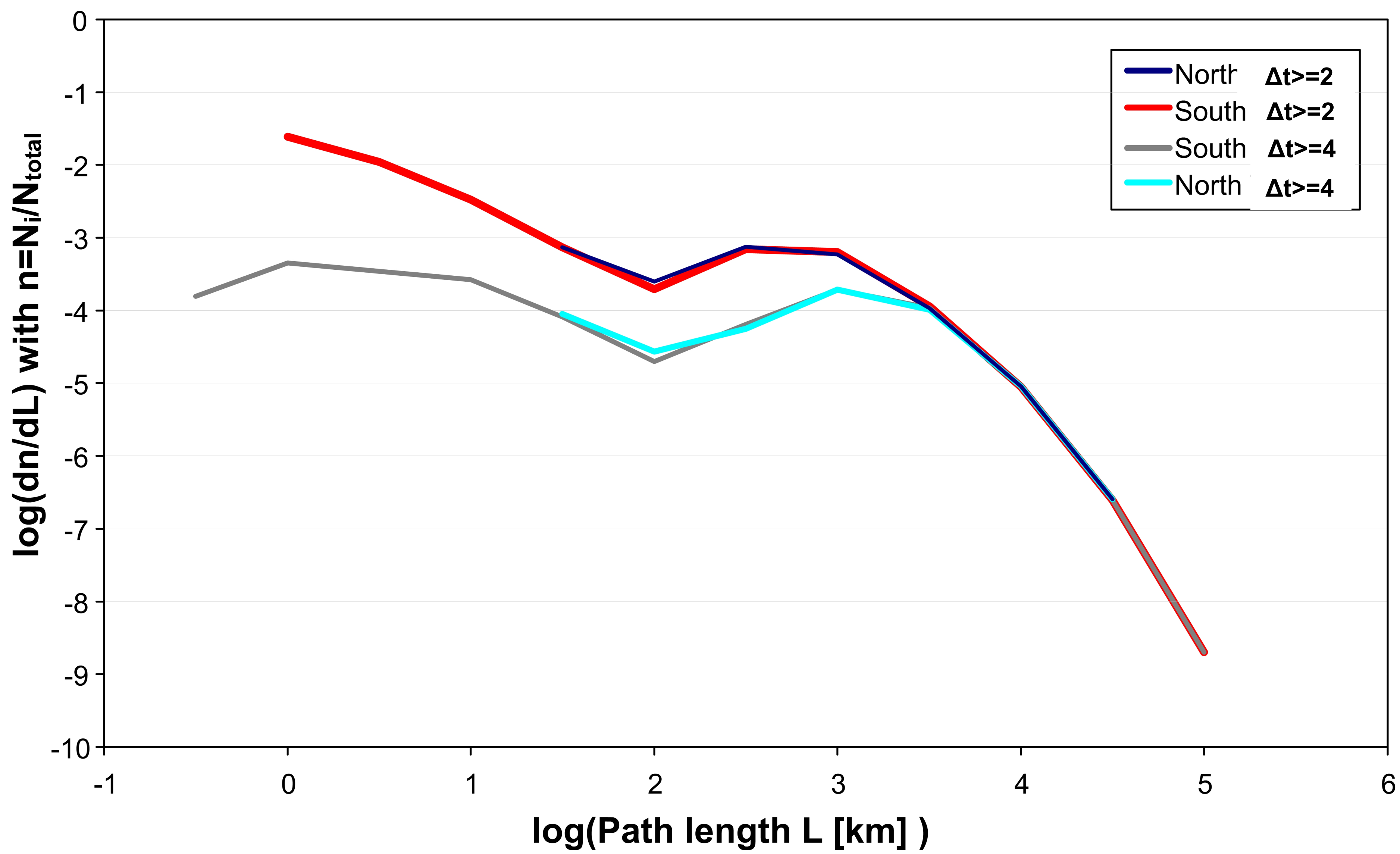


Area



N_{total} (South) = 60247
N_{total} (North) = 23251 ,
log(dn/dA) with n=Ni/Ntotal
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
North all
South all
North Δt>=2
South Δt>=2
North Δt>=4
South Δt>=4
-2
-1,5
-1
-0,5
0
0,5
1
1,5
2
2,5
3
3,5
4
4,5
5
5,5
6
6,5
7
log(A[km²])

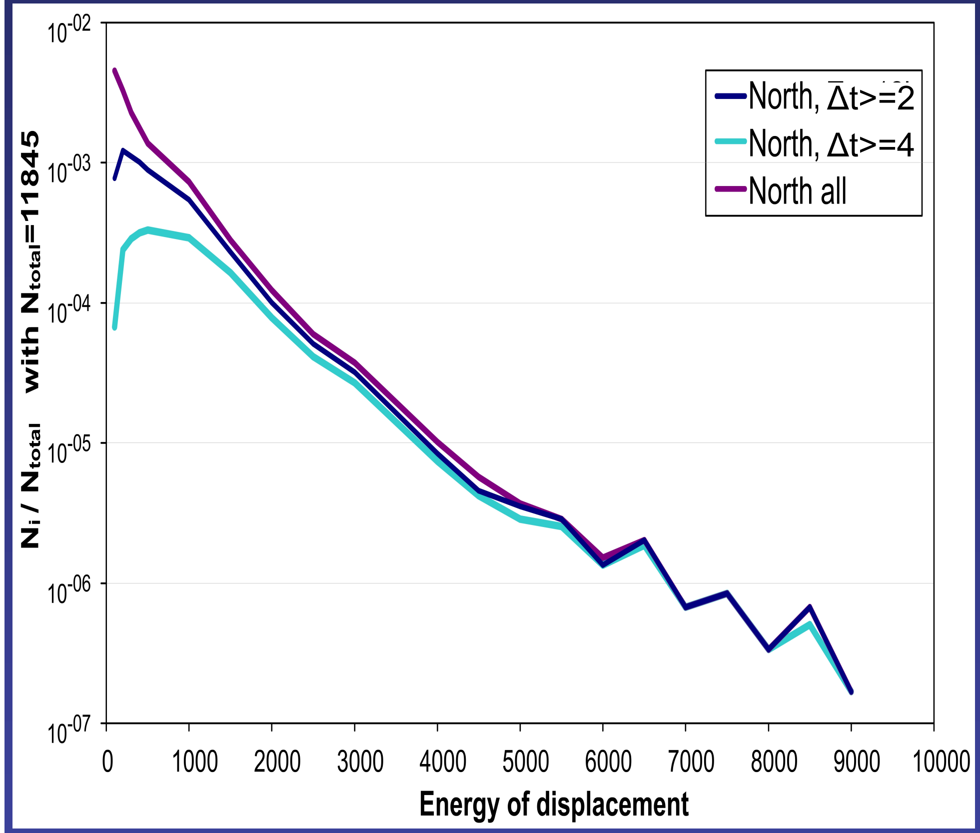
Path length



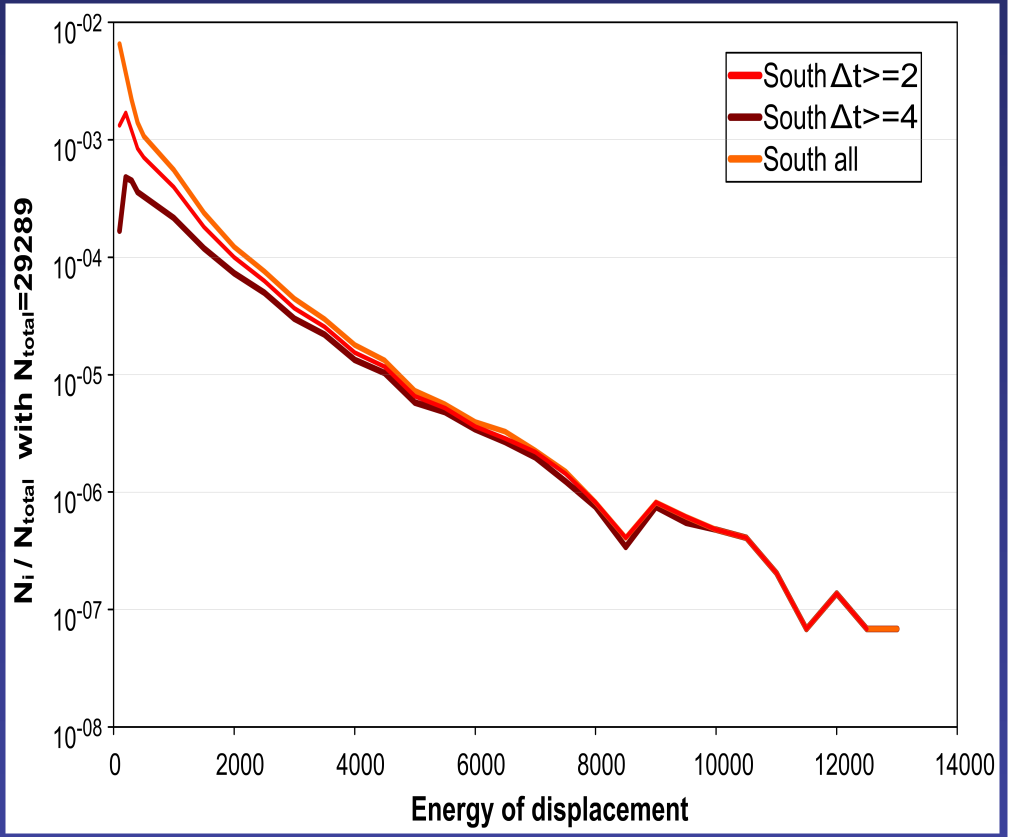
North Δt>=2
South Δt>=2
South Δt>=4
North Δt>=4
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-1
0
1
2
3
4
5
6
log(dn/dL) with n=Ni/Ntotal
log(Path length L [km])

Local maximum intensity

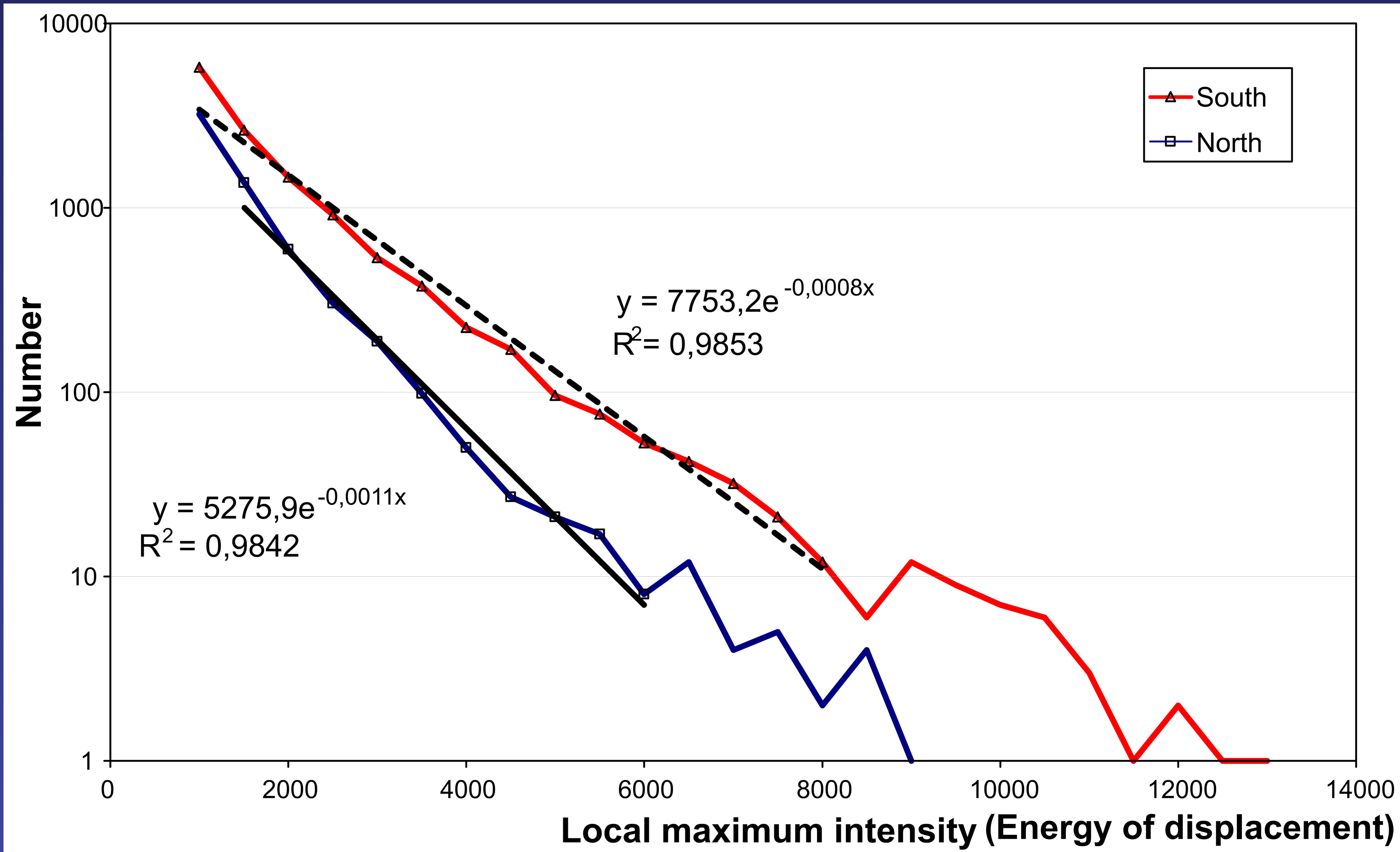
Northern Hemisphere



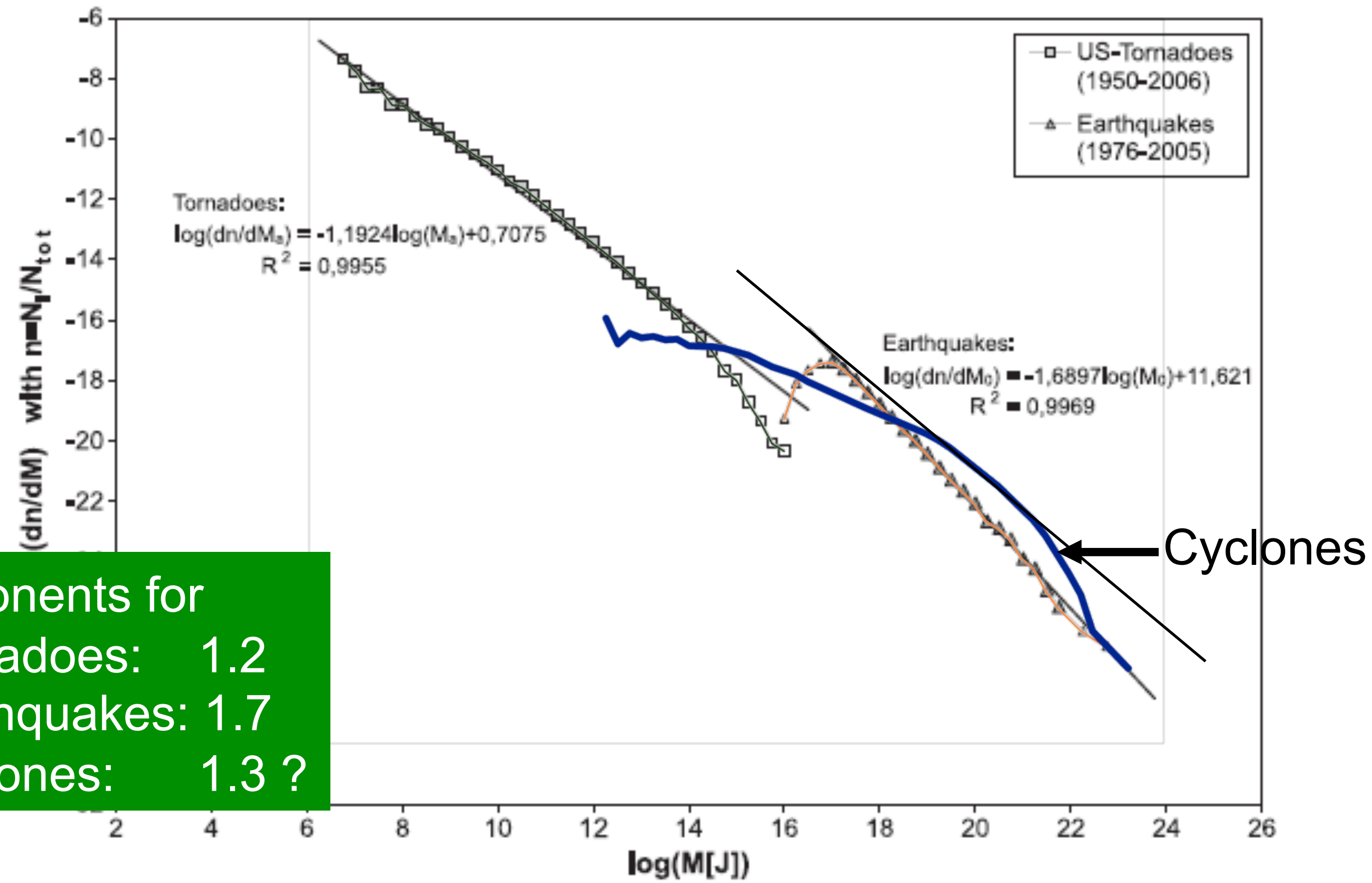
Southern Hemisphere



Comparison N/S-hemisphere intensity



Results



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