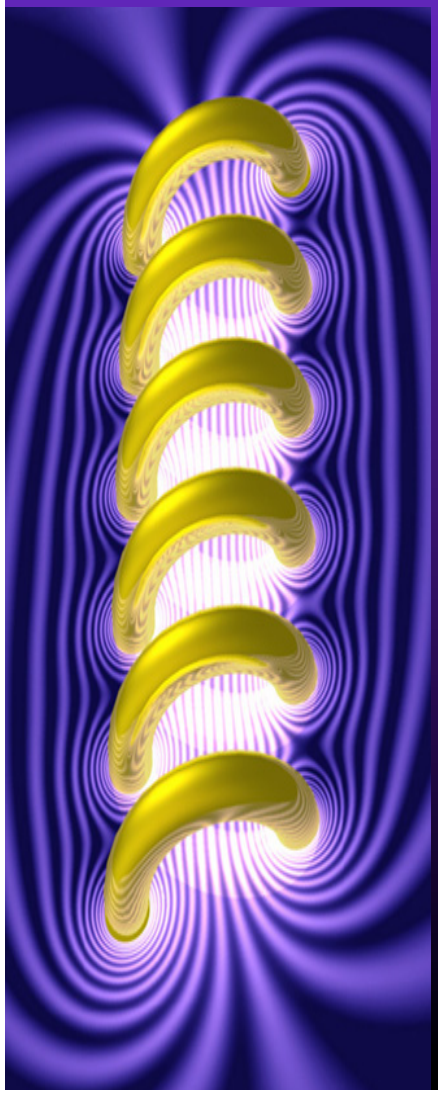
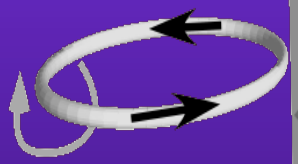
Vorticity induction: alignment of RFD vorticity within the mesocyclone

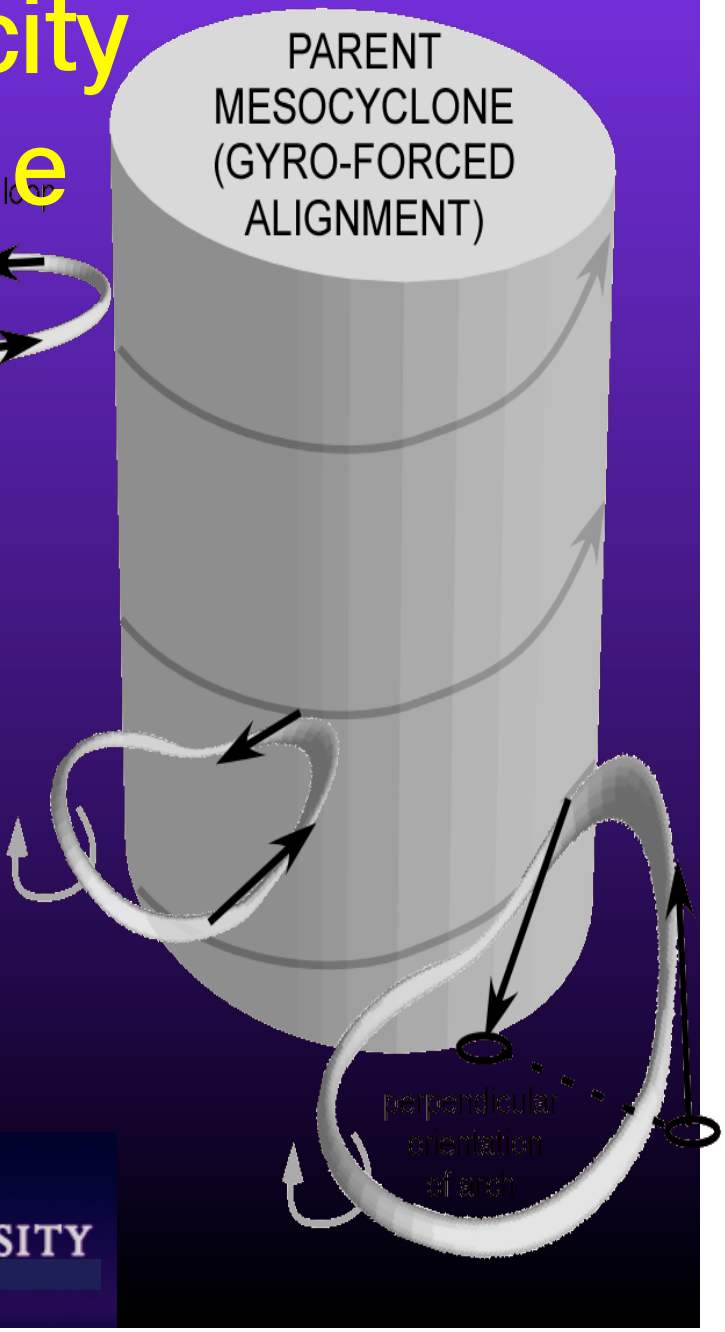
ECSS - 2011



-Or-



Vorticity and scale interaction using the electromagnetic-hydrodynamic analogy



Marcus Büker

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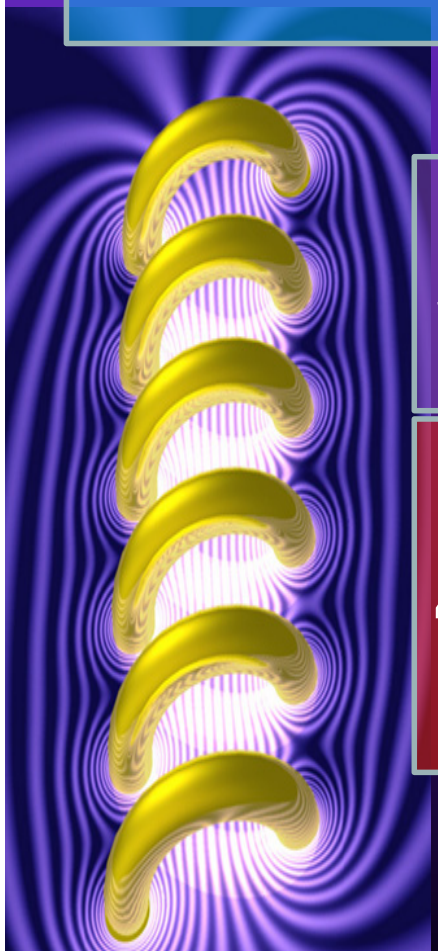
1 **Vorticity induction:**

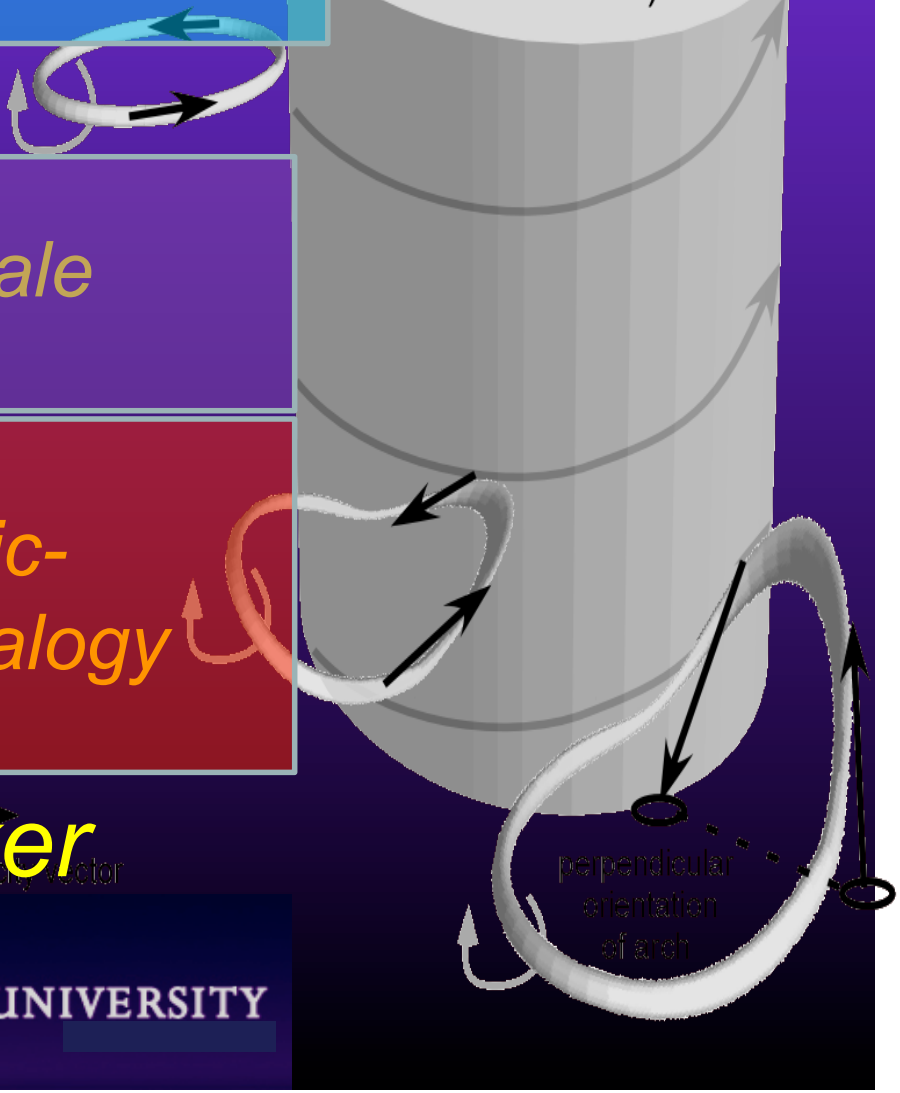
2 alignment of RFD vorticity within the mesocyclone

-Or-

3 *Vorticity and scale interaction*

4 *using the electromagnetic-hydrodynamic analogy*





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GLOSSARY OF METEOROLOGY

magnetic induction:

A vector field, usually denoted by B, defined as follows:

The torque N experienced by a magnetic dipole with magnetic dipole moment m is

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}.$$

Thus by measuring N for m oriented in two orthogonal directions, the magnetic induction components are obtained as torque components divided by the magnitude of m.

Major disclaimer # 1

- Although there have been some studies investigating actual electrical effects on tornadogenesis…
- This work is only examining the **analogous behavior** between electromagnetism and fluid dynamics.
- **In other words, we are NOT claiming electromagnetic forces are dominant.**

1 **Vorticity induction:**

2 alignment of RFD vorticity within the mesocyclone

-Or-

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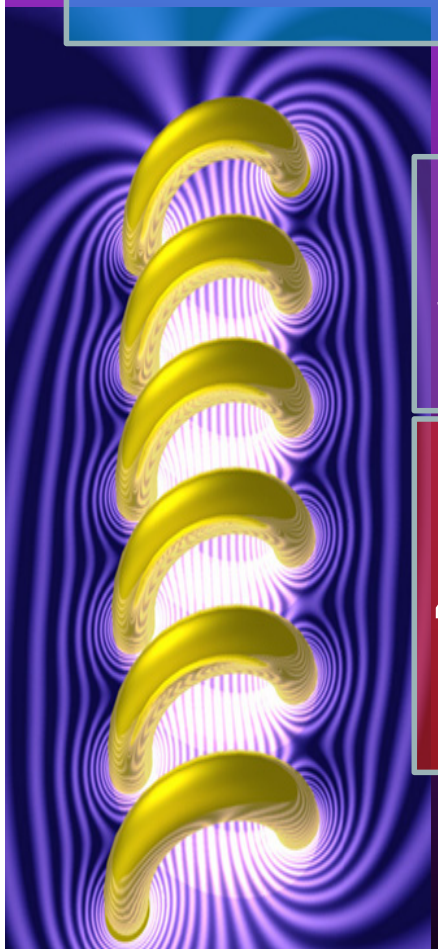

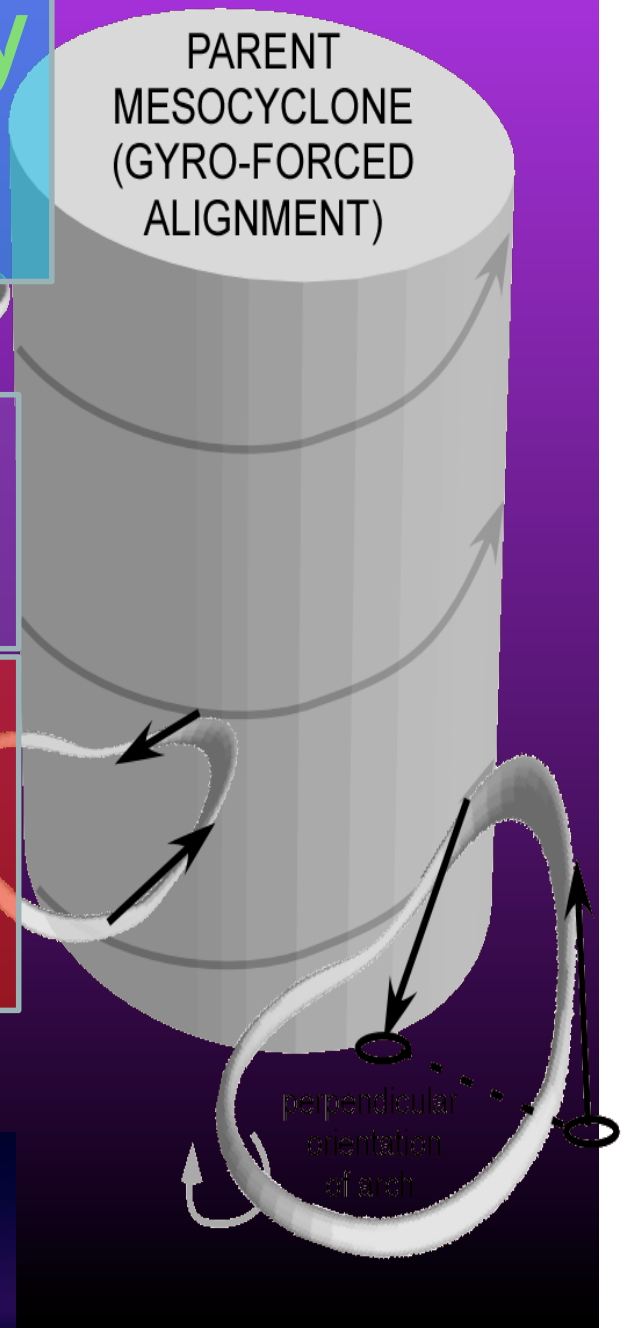

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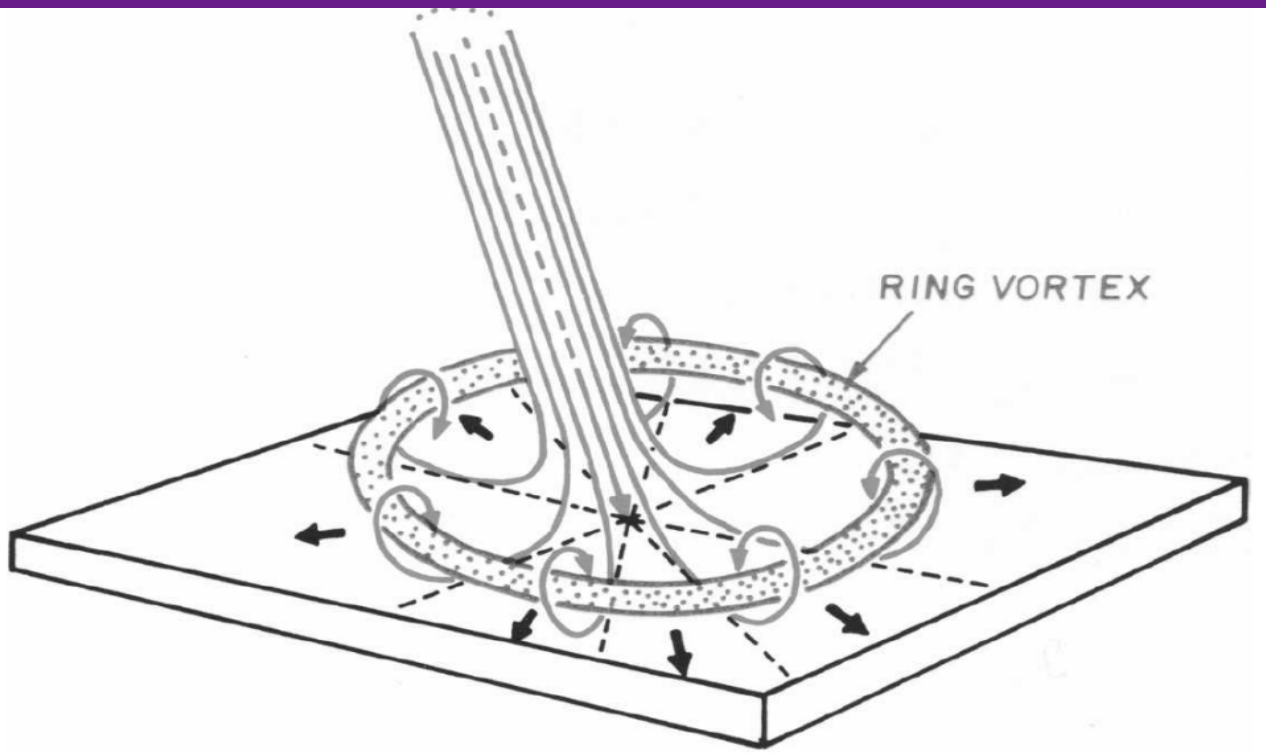
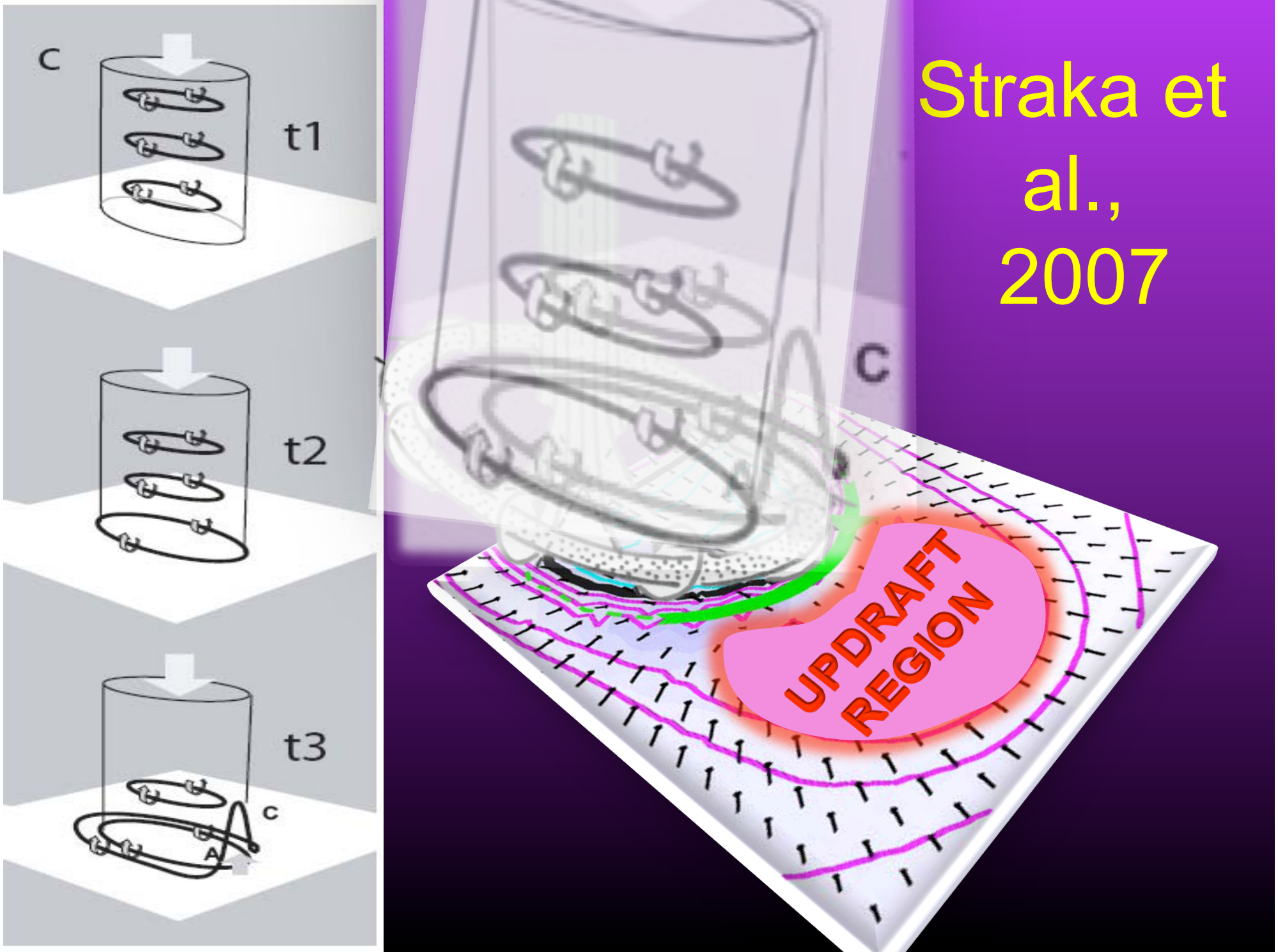
2 alignment of RFD vorticity within the mesocyclone

- The rear flanking downdraft (RFD) region in a supercell provides a baroclinic source of quasi-horizontal vorticity

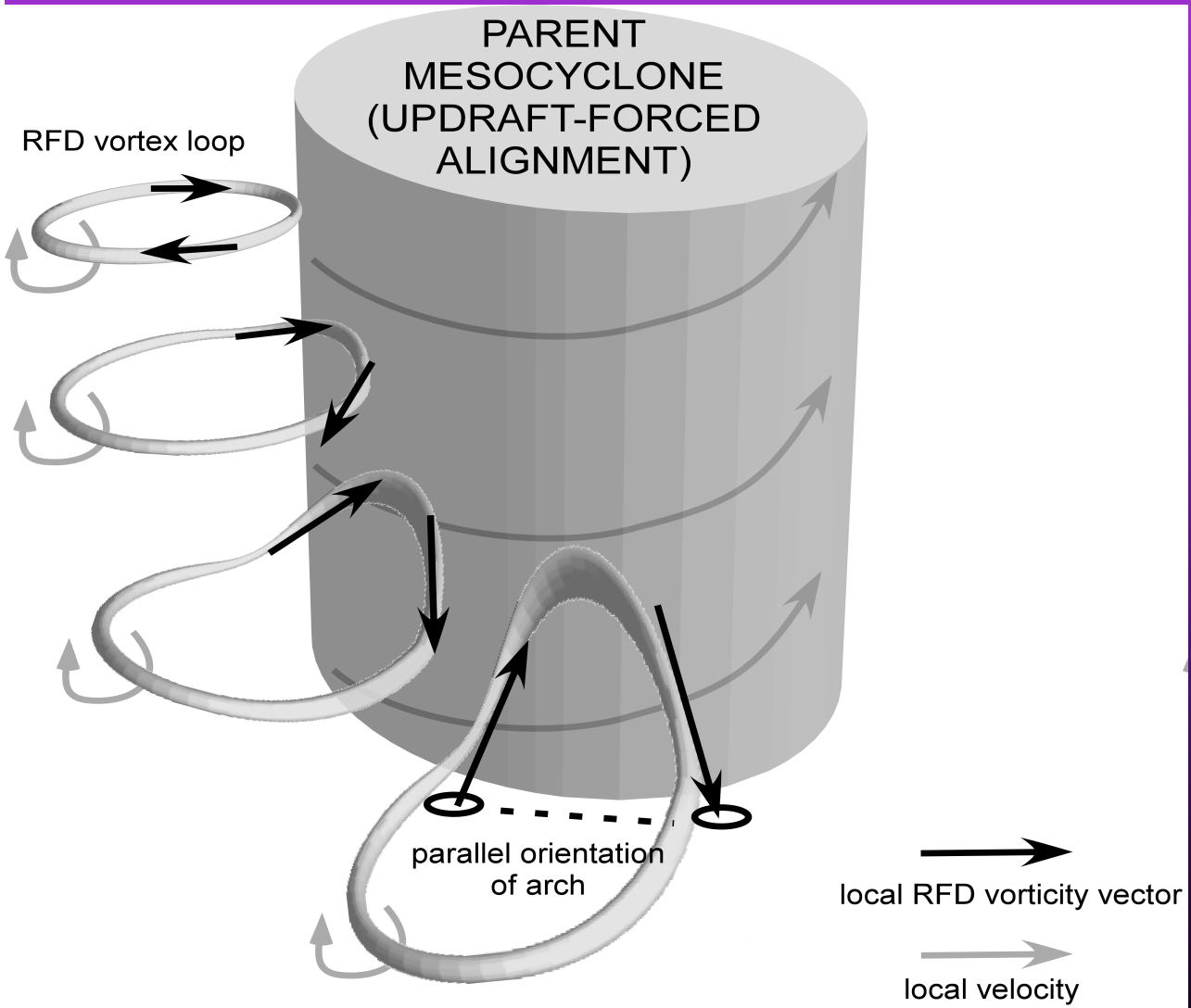
Figure:
Fujita (1985)



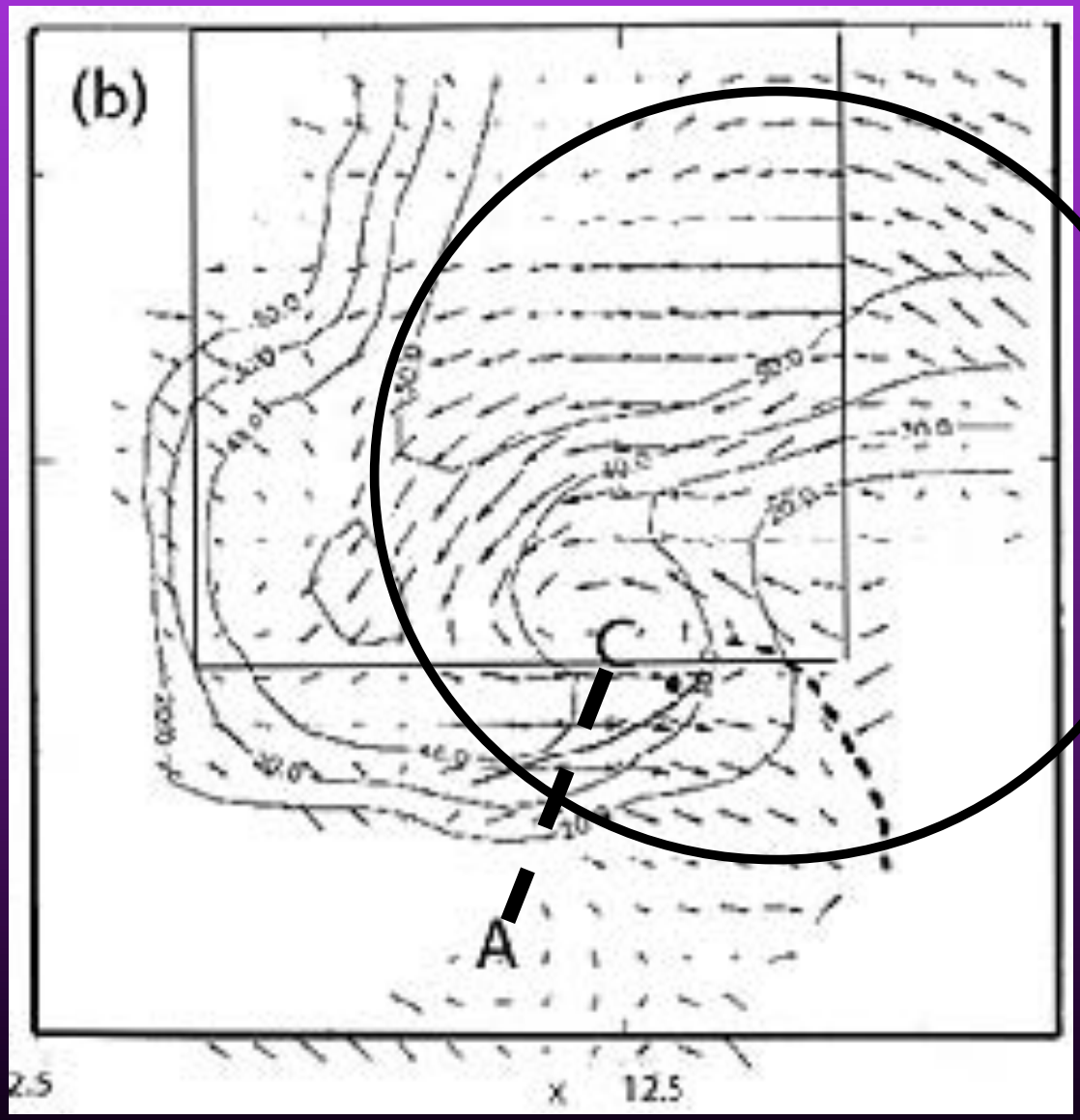


Straka et al., 2007

If the strongest updraft is in the vicinity of the main mesocyclone, one would expect that a propagating RFD vortex ring typically would be lifted quasi-symmetrically into a parallel arch structure.



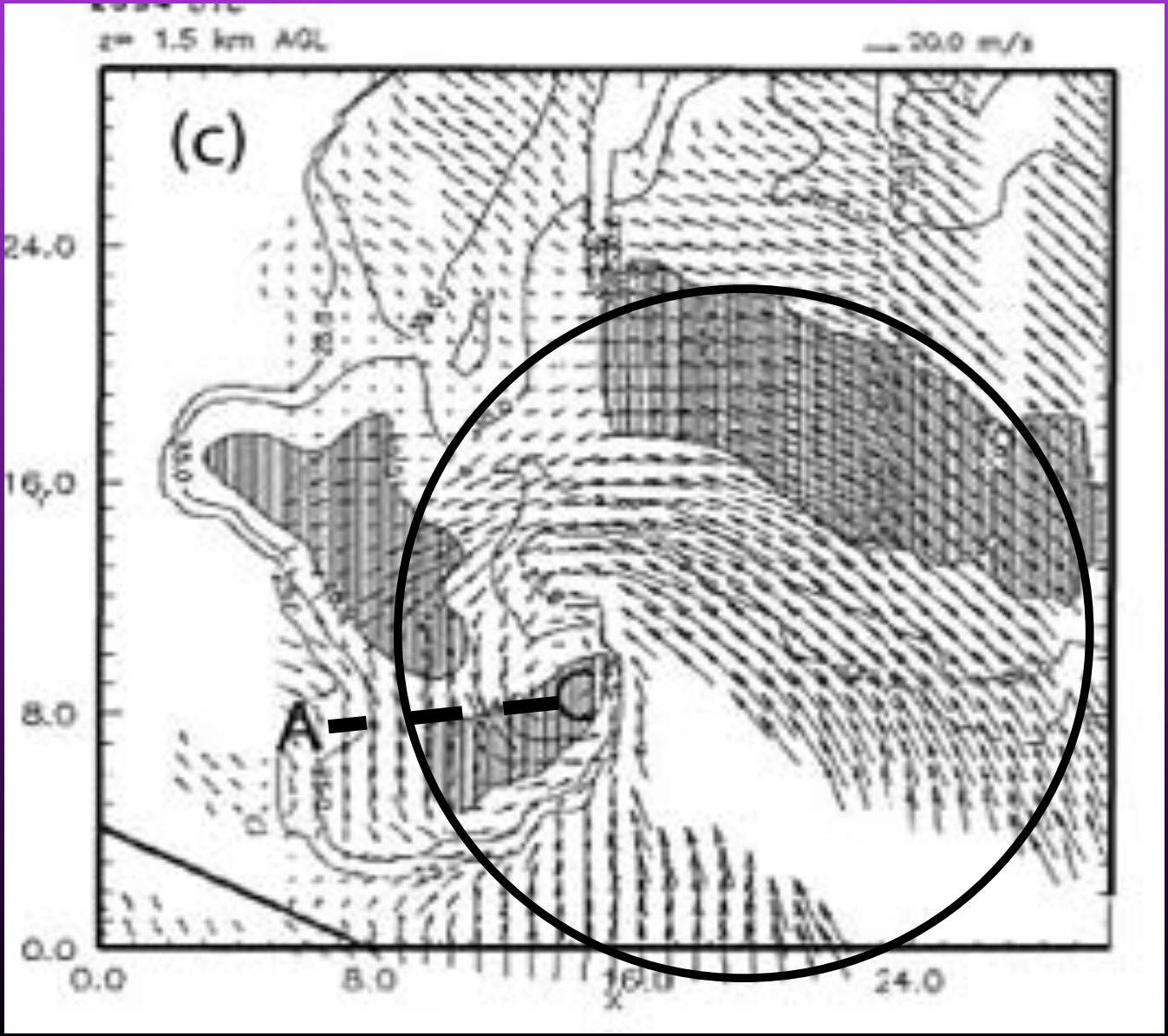
Observations (Straka et al., 2007)



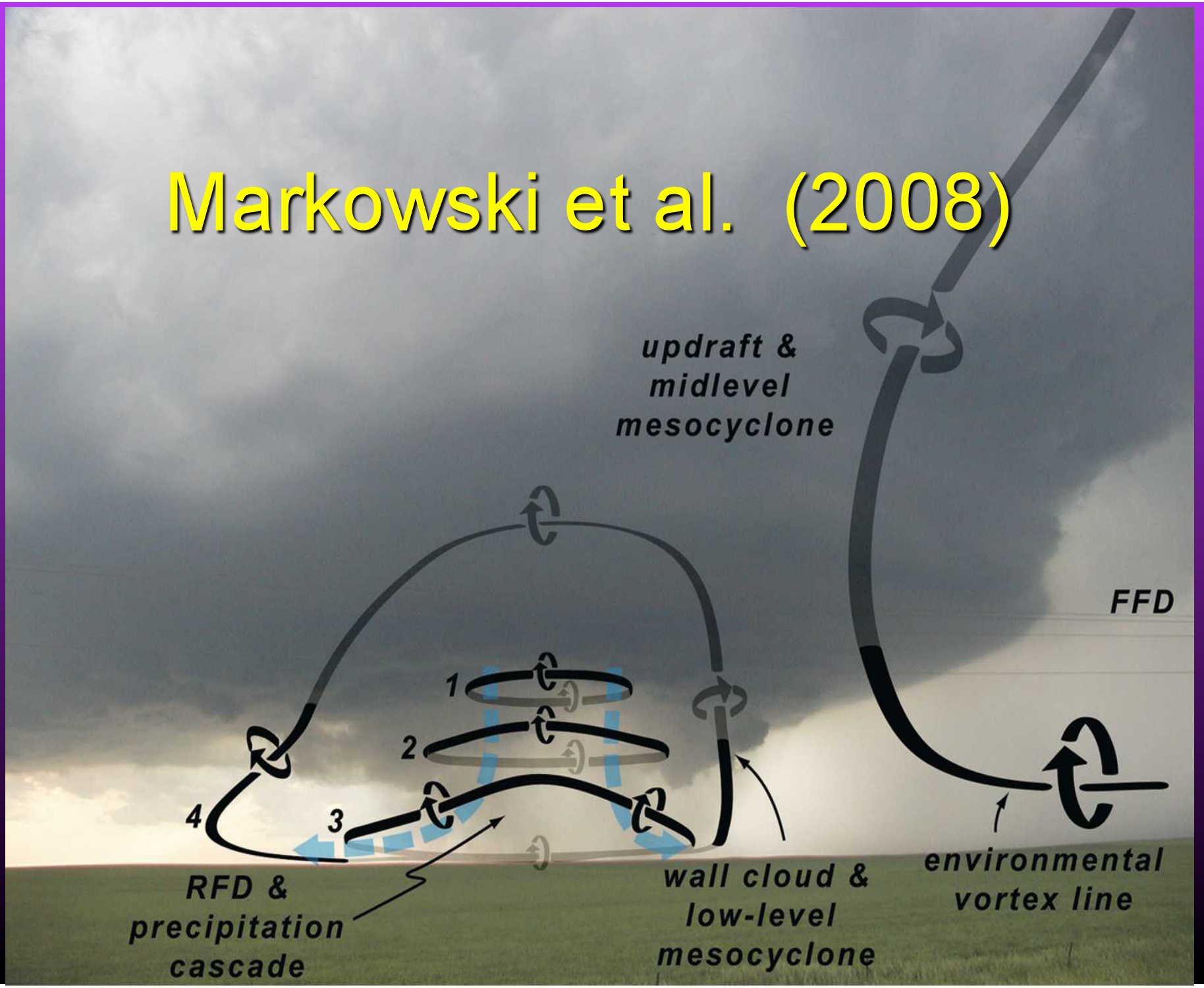
A-C pair =

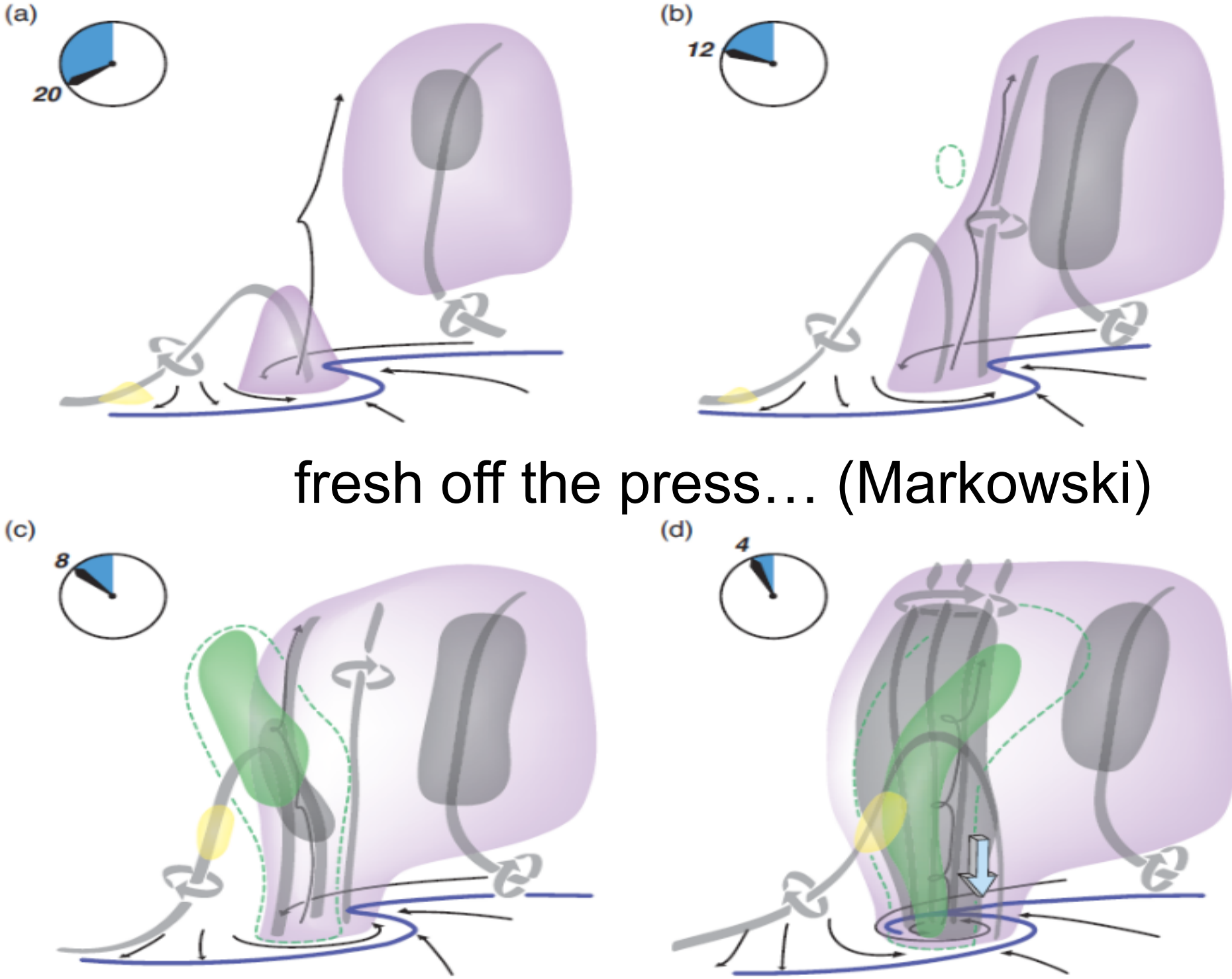
Anticyclonic-Cyclonic Vortex couplet (arch)

Observations (Straka et al., 2007)



Markowski et al. (2008)



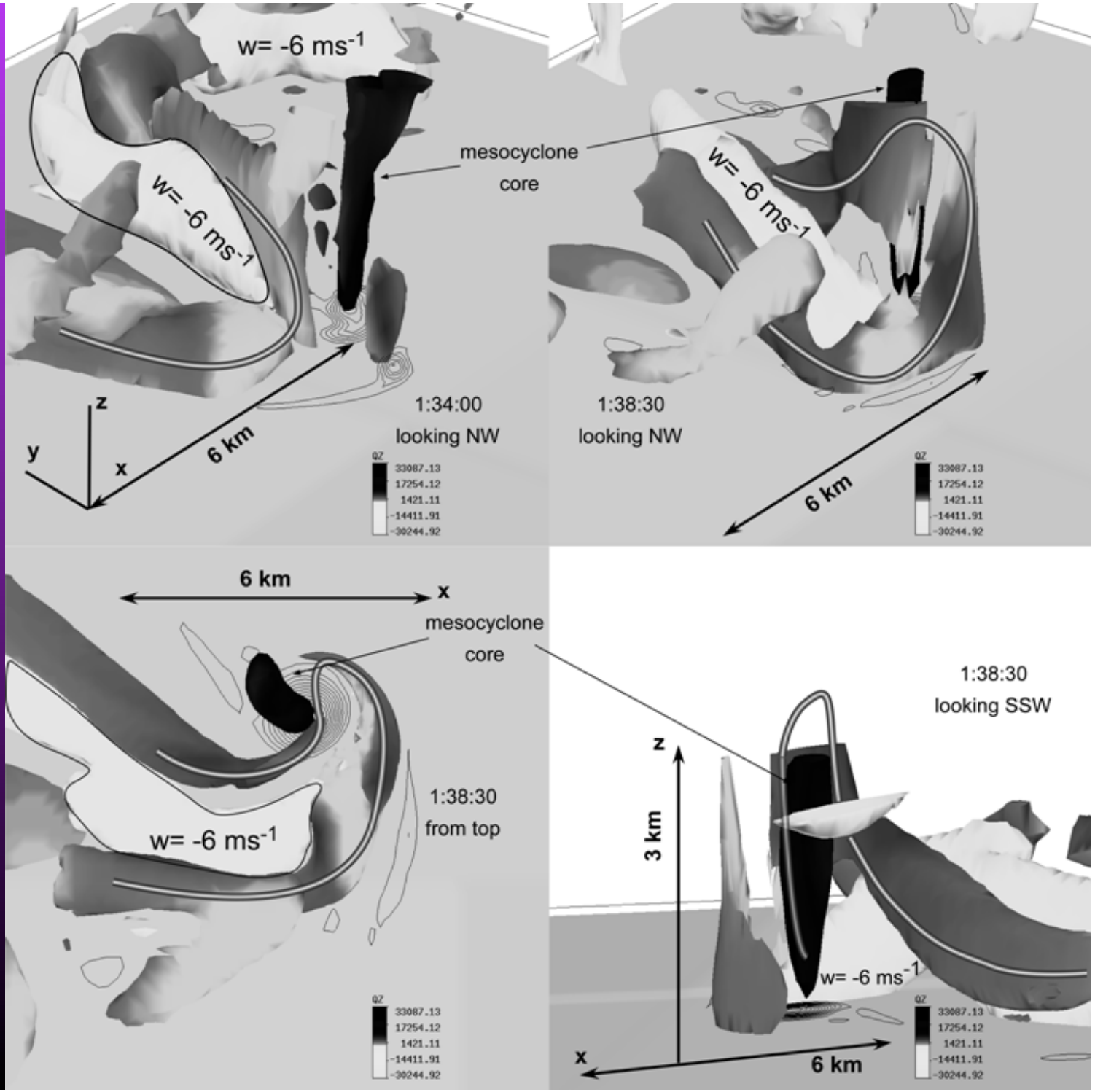


fresh off the press… (Markowski)

UWNMS tornadic supercell simulations

Initial conditions: Gaudet and Cotton (2006)

120 meter resolution on 3rd grid



- ***COULD THIS JUST BE DIFFERENTIAL LIFTING?***

- ***Perhaps…***

- ***But why do observations of the CAC (cyclonic/anticyclonic couplet) show such asymmetry and a consistent perpendicular orientation to the mesocyclone axis?***

- ***When relatively strong, unaligned vorticity with disparate spatial scales becomes concentrated in a small location, this looks like a rather interesting problem in***

3 *Vorticity and scale interaction*

- ***If we isolate the inertial (vortex-vortex) nonlinear scale interactions, the features start resembling the building blocks of inhomogeneous turbulence.***

Major disclaimer # 2

- We are not claiming that the following mechanisms are the only way that RFD induced horizontal vorticity becomes vertically aligned…
- …only that there are aspects of multi-scale vortex interaction that may need to be closely considered as being part of the complex tornadogenesis dynamics chain.

4 *The electromagnetic-hydrodynamic (EM-HD)*

analogy has been established in fluid dynamics literature, used specifically to attack problems involving , for example:

- ***turbulence (Marmanis, 2000),***
- ***vortex tracking (Rosseaux, 2006),***
- ***generalized, non-relativistic causal fluids (Belevich, 2008).***

- ***The electromagnetic-hydrodynamic (EM-HD) analogy is not a new concept:*** origins extend at least back to the works of **Helmholtz (1858)** and **Maxwell (1861)**

As an aside, this analogy is also being used in theoretical physics to further advance understanding of electromagnetism…

e.g. Mario Pinheiro

Instituto de Plasmas e Fusão Nuclear (IPFN)

Instituto Superior Técnico - Lisbon

- ***Belevich (2008): the mathematical similarity between hydrodynamics and electromagnetism is VERY extensive***

‘Traditional’ EM-HD analogy

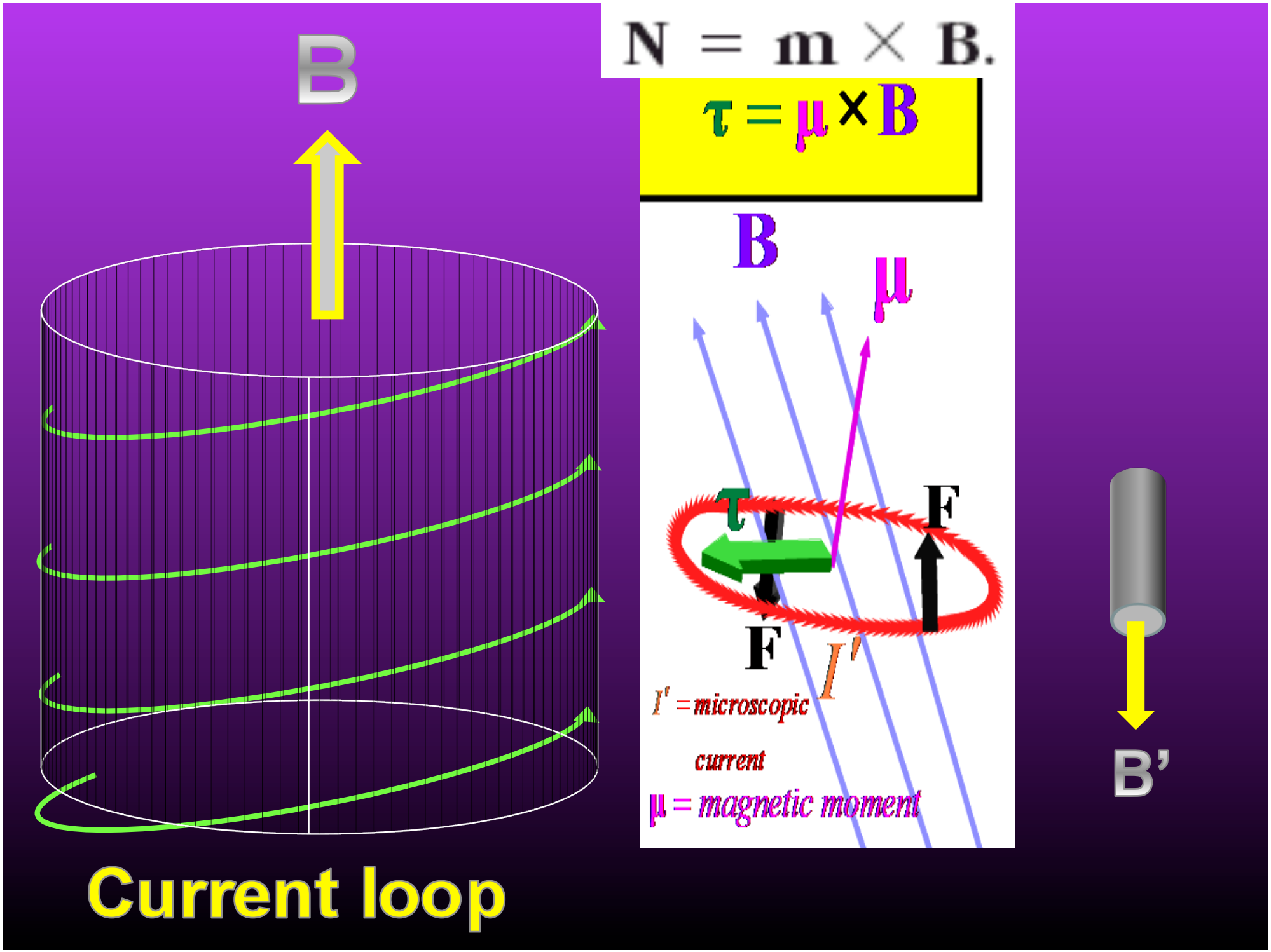
Turbulent hydrodynamics	Electromagnetism (after Marmanis(1998), Belevich(2008) and Pinheiro (2009))	Analogous Variables
Navier-Stokes $\frac{\partial \mathbf{u}}{\partial t} = -(\boldsymbol{\omega}\times\mathbf{u}) - \nabla\left(\frac{p}{\rho} + \frac{u^2}{2}\right) + \nu\nabla^2\mathbf{u}$	$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\phi$ Vector and scalar potential $\mathbf{E}$ = electric field	$\mathbf{u}$ $\mathbf{A}$ $\mathbf{l}$ $\mathbf{E}$
Lamb vector and vorticity $\mathbf{l} \equiv (\boldsymbol{\omega}\times\mathbf{u})$ $\nabla\cdot\boldsymbol{\omega} = 0$ $\boldsymbol{\omega} = \nabla\times\mathbf{u}$	$\mathbf{B} = \nabla\times\mathbf{A}$ $\nabla\cdot\mathbf{B} = 0$ Coloumb Thomson magnetic induction	$\boldsymbol{\omega}$ $\mathbf{B}$
vorticity tendency $\frac{\partial \boldsymbol{\omega}}{\partial t} = -\nabla\times\mathbf{l}^* + \nu\nabla^2\boldsymbol{\omega}$ • viscous term removed	$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$ Faraday's Law	
Lamb vector tendency and turbulent current ($\mathbf{j}$) $\frac{\partial \mathbf{l}}{\partial t} = \nabla\times\boldsymbol{\eta} - \mathbf{j}$	$\frac{\partial(\varepsilon_0\mathbf{E})}{\partial t} = c^2\nabla\times\mathbf{H} - \mu_0\mathbf{J}$ Ampere's Law (zero polarization)	$\boldsymbol{\eta}$ $\mathbf{H}$
vorticity field strength ($\boldsymbol{\eta}$) and magnetization $\boldsymbol{\eta} = \lvert u^2\rvert\boldsymbol{\omega} - \mathbf{M}$ $\mathbf{M} = \mathbf{u}(\mathbf{u}\cdot\boldsymbol{\omega}) + \nu\nabla^2\mathbf{u}$	$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$ magnetic field strength and magnetization	
turbulent charge density $\nabla\cdot\mathbf{l} = \mathbf{u}\cdot\nabla\times\boldsymbol{\omega} - \lvert\omega^2\rvert \equiv \rho_n$	$\nabla\cdot(\varepsilon_0\mathbf{E}) = \rho_e$ electric charge density	ρ_n ρ_e

To date, these efforts have utilized the analogy in either a kinematic, diagnostic, or statistical sense…

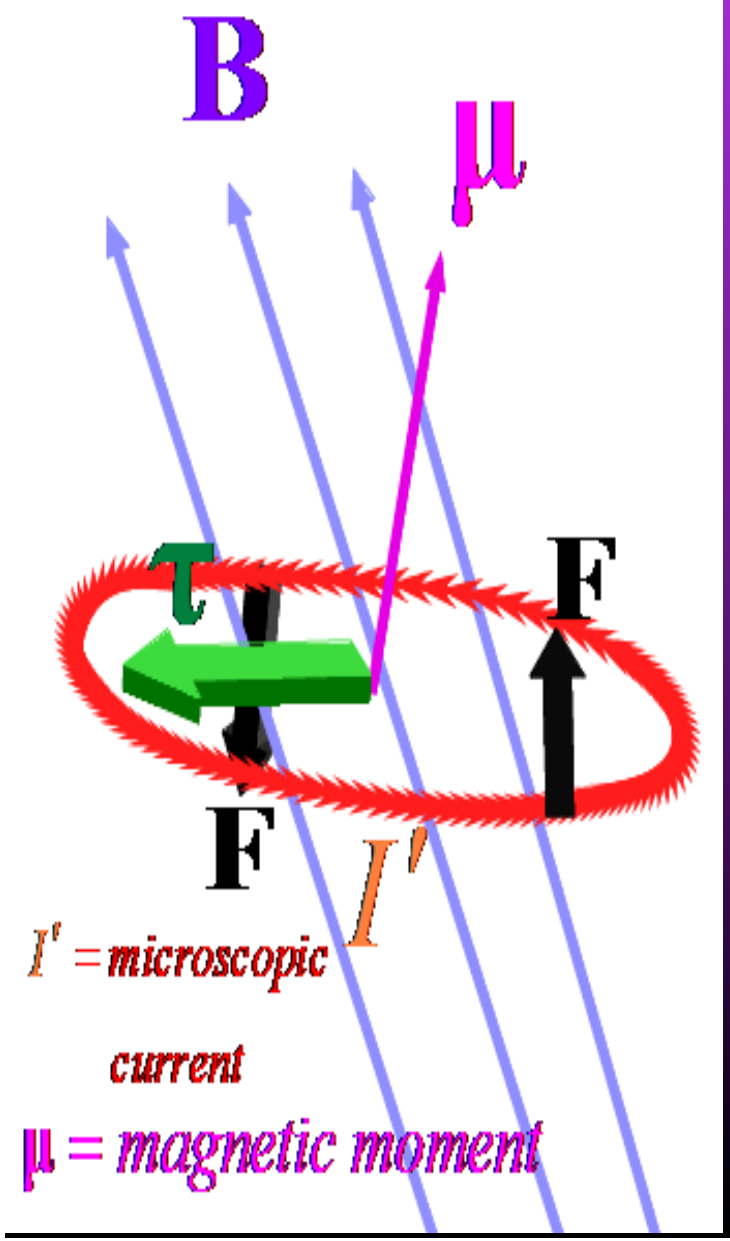
…but if the mathematics are so deeply similar, perhaps the analogy can extend dynamically as well.

A smaller summary…

Electromagnetism	Hydrodynamics
Vector Potential $\mathbf{A}$	Fluid velocity $\mathbf{u}$
Magnetic Induction $\mathbf{B} = \nabla \times \mathbf{A}$	Vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$
Electric Field $\mathbf{E}$	Lamb vector $\mathbf{l} = \boldsymbol{\omega} \times \mathbf{u}$
Charge density $\Xi = \nabla \cdot \mathbf{E}$	"Turbulent charge" $n = \nabla \cdot \mathbf{l}$

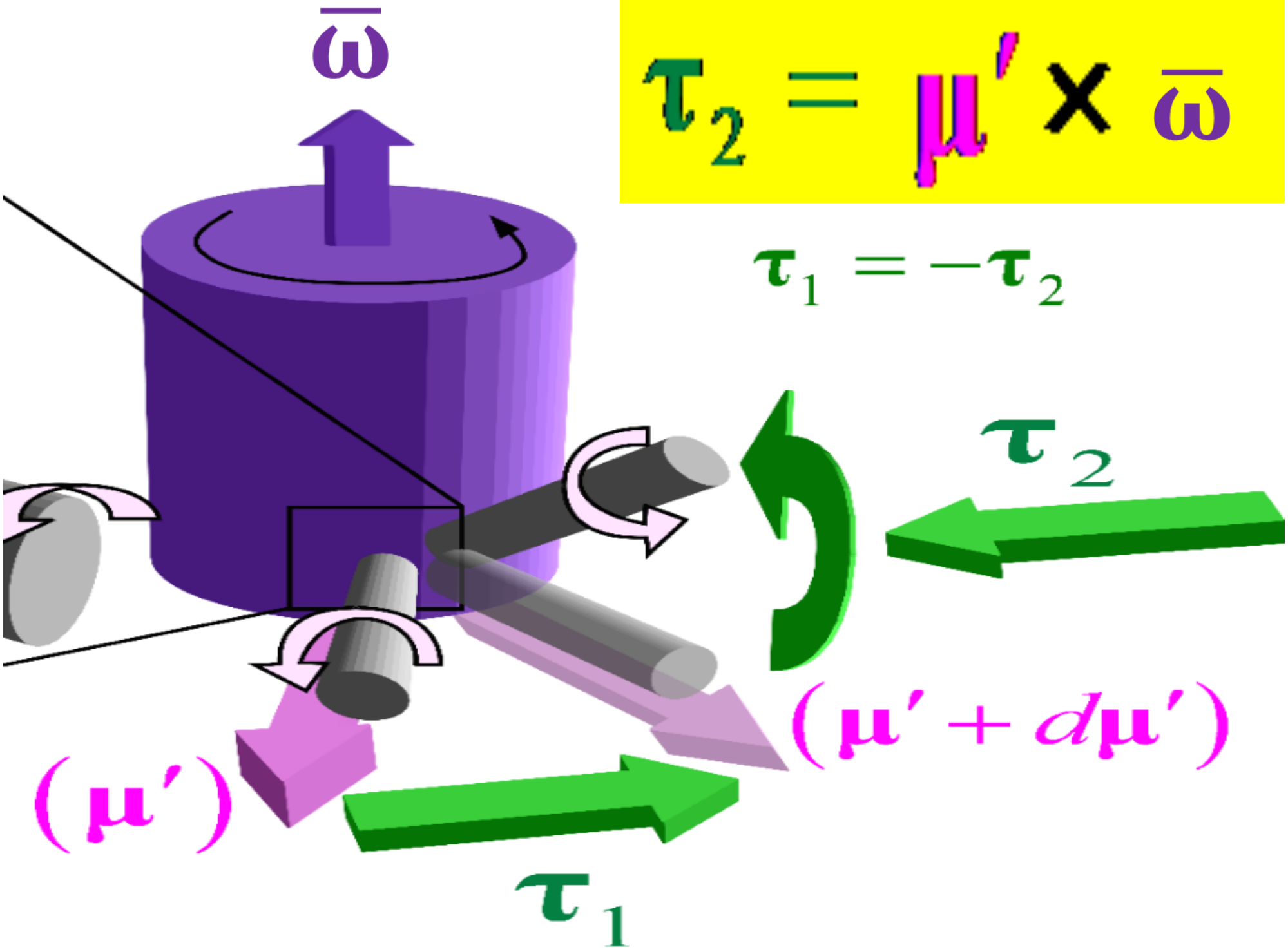

B
N = m × B.
τ = μ × B
B
μ
τ
F
F
I′
I′ = microscopic current
μ = magnetic moment
B’
Current loop

$$\tau = \mu \times B$$



$$\tau_2 = \mu' \times \bar{\omega}$$

$$\mu = r \times \rho u$$


$\bar{\omega}$
$\tau_2 = \mu' \times \bar{\omega}$
$\tau_1 = -\tau_2$
τ_2
$(\mu' + d\mu')$
(μ')
τ_1

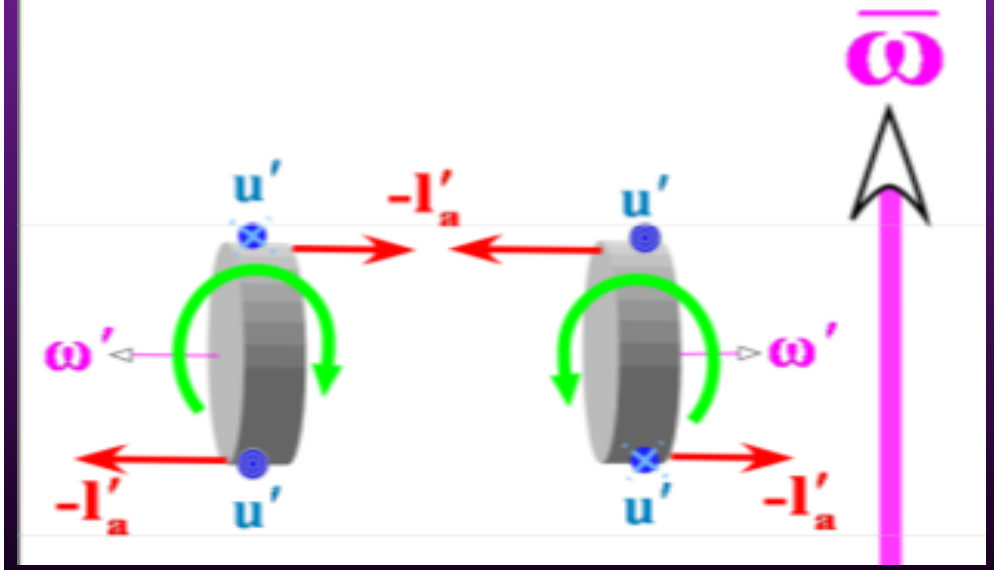
Mathematically…

Lamb vector curl

$$\iota \quad \tau\tau_{HH} \Big/ I \Big/ |\mathbf{r}|^2 \Big/ HH \quad -$$

$$(\bar{\omega}\times\bar{u}) + (\bar{\omega}\times u') + (\omega'\times\bar{u}) + (\omega'\times u')$$

$$\bar{l} \qquad l'_a \qquad l'_b \qquad l''$$



"proof left to the viewer as an exercise…"

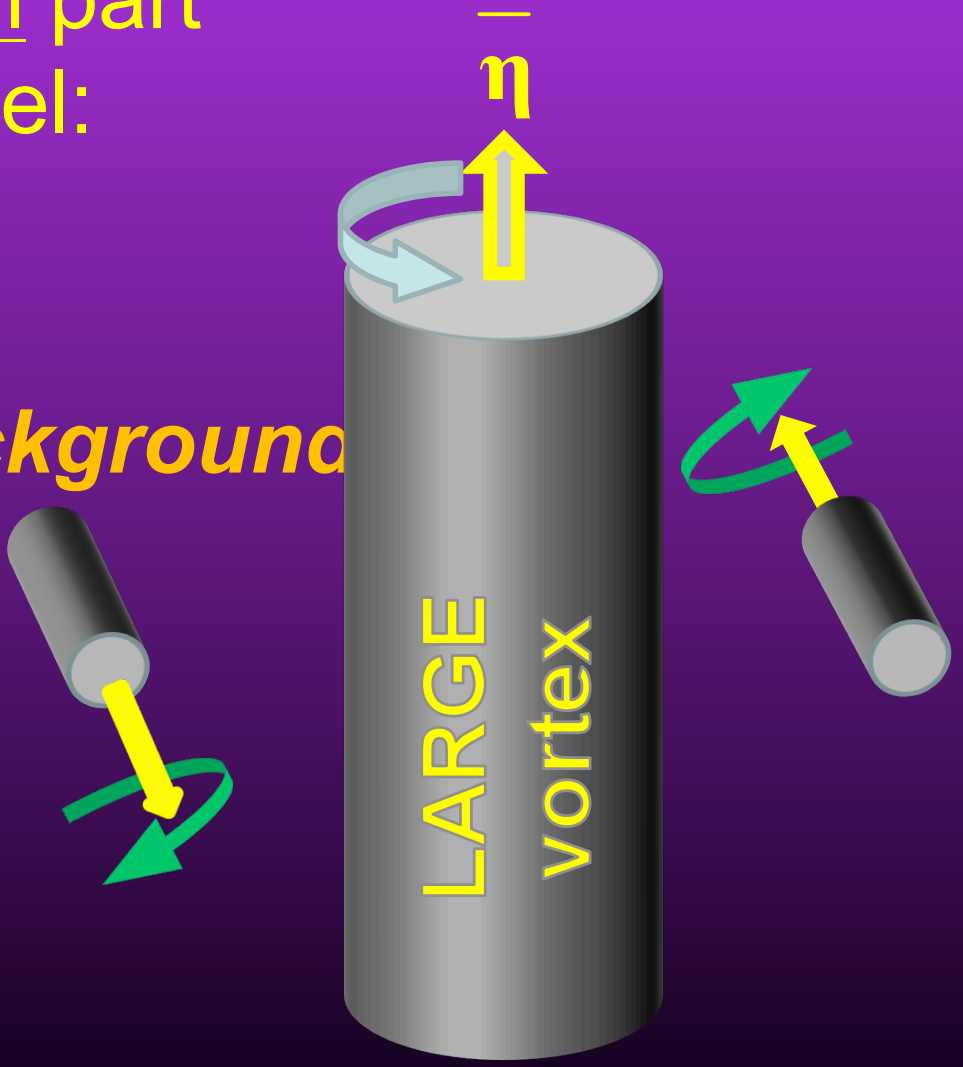
…or see our *(hopefully soon to be finished)* paper

Idealized vortex simulations

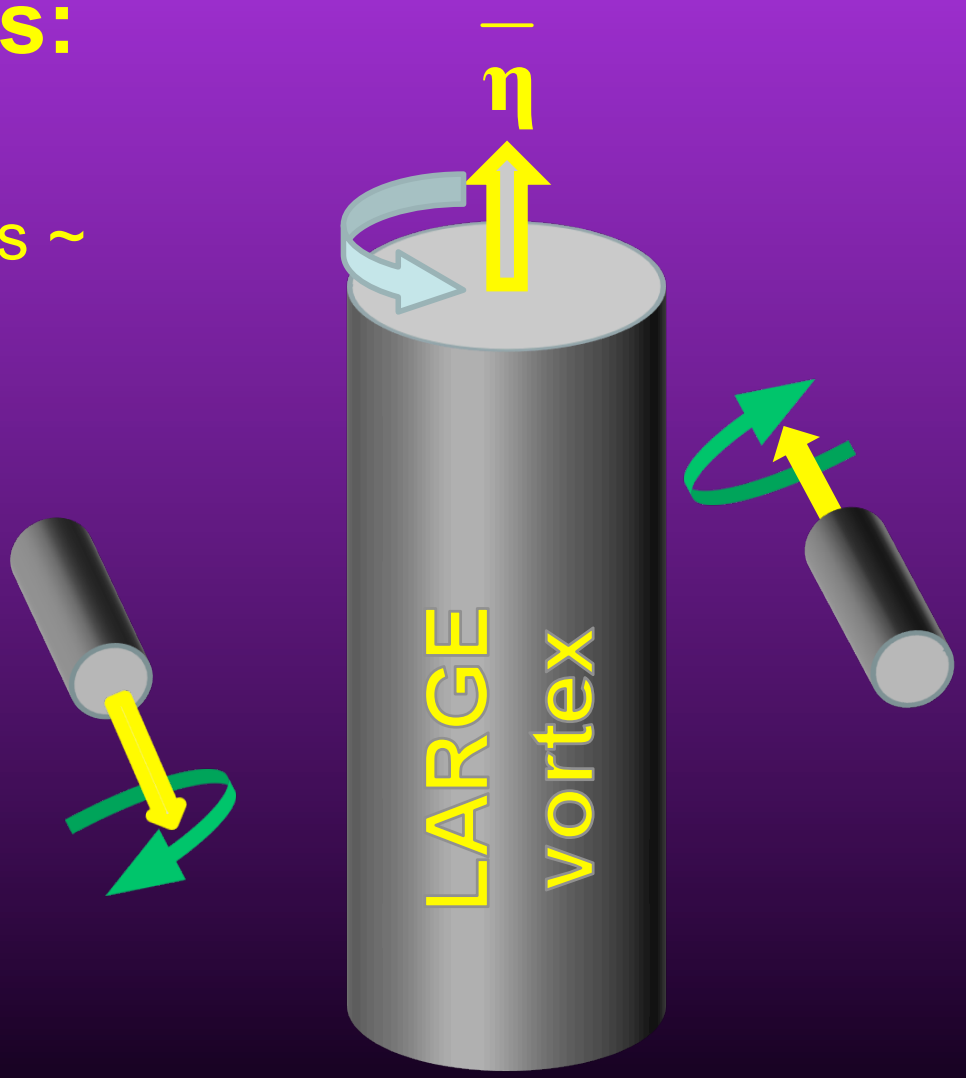
- Initial test for this small part of the conceptual model:

  Will small-scale vortices naturally align with a background vorticity field?

- UW-NMS (Tripoli, 1992)
- dx=dy=dz=120m
- 200 x 200 x 100 pts

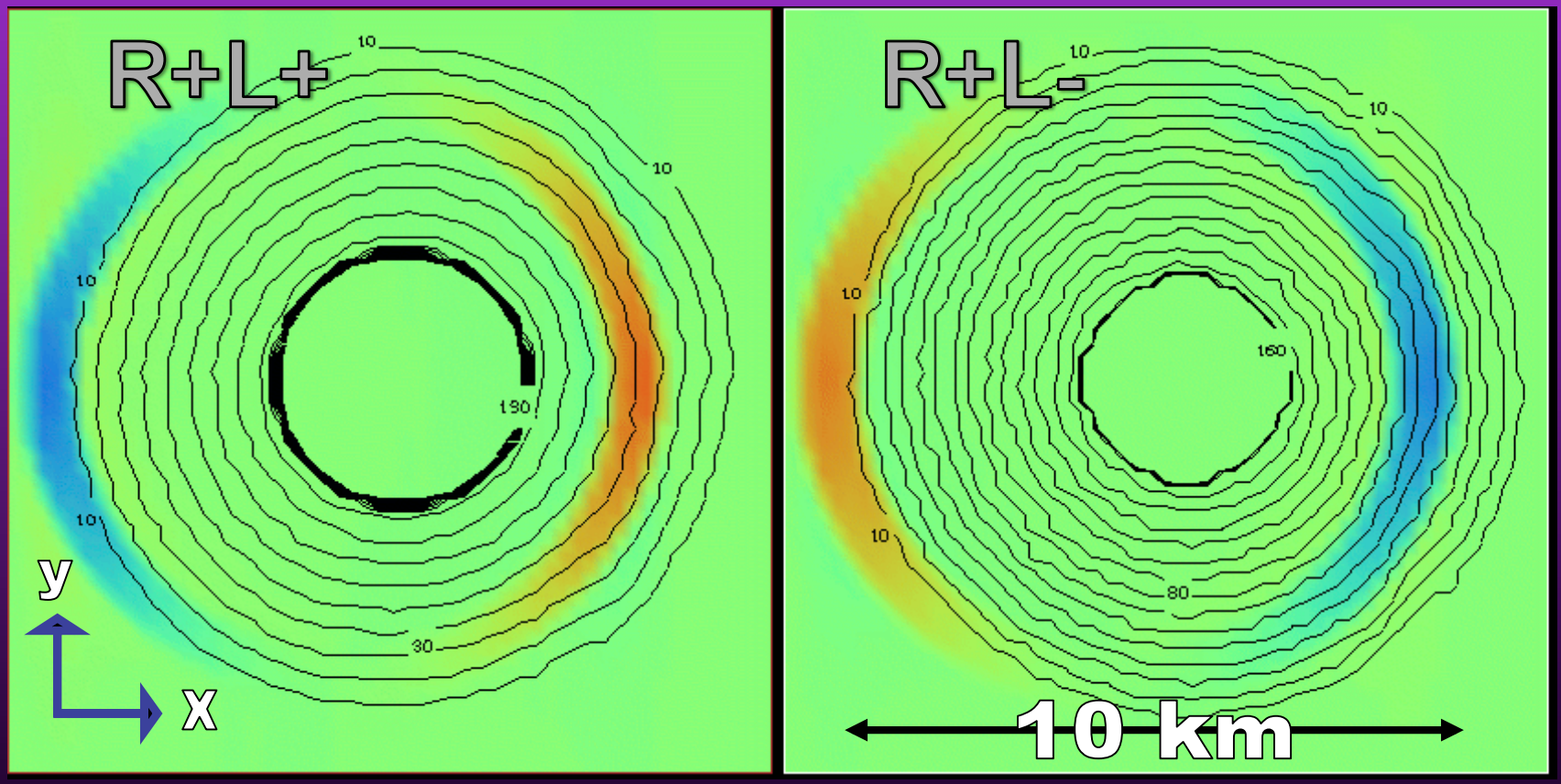


- Neutral stability: **no vertical forcing or thermal perturbations: NO UPDRAFT**
- Vertical Rankine vortex, radius ~ 5 km, surrounded by ring of opposite signed vorticity
- Surrounding vortex loop of horizontal vorticity, varying in magnitude
- **Total initialized vorticity = 0**
- Pressure field balanced using 3-D solver



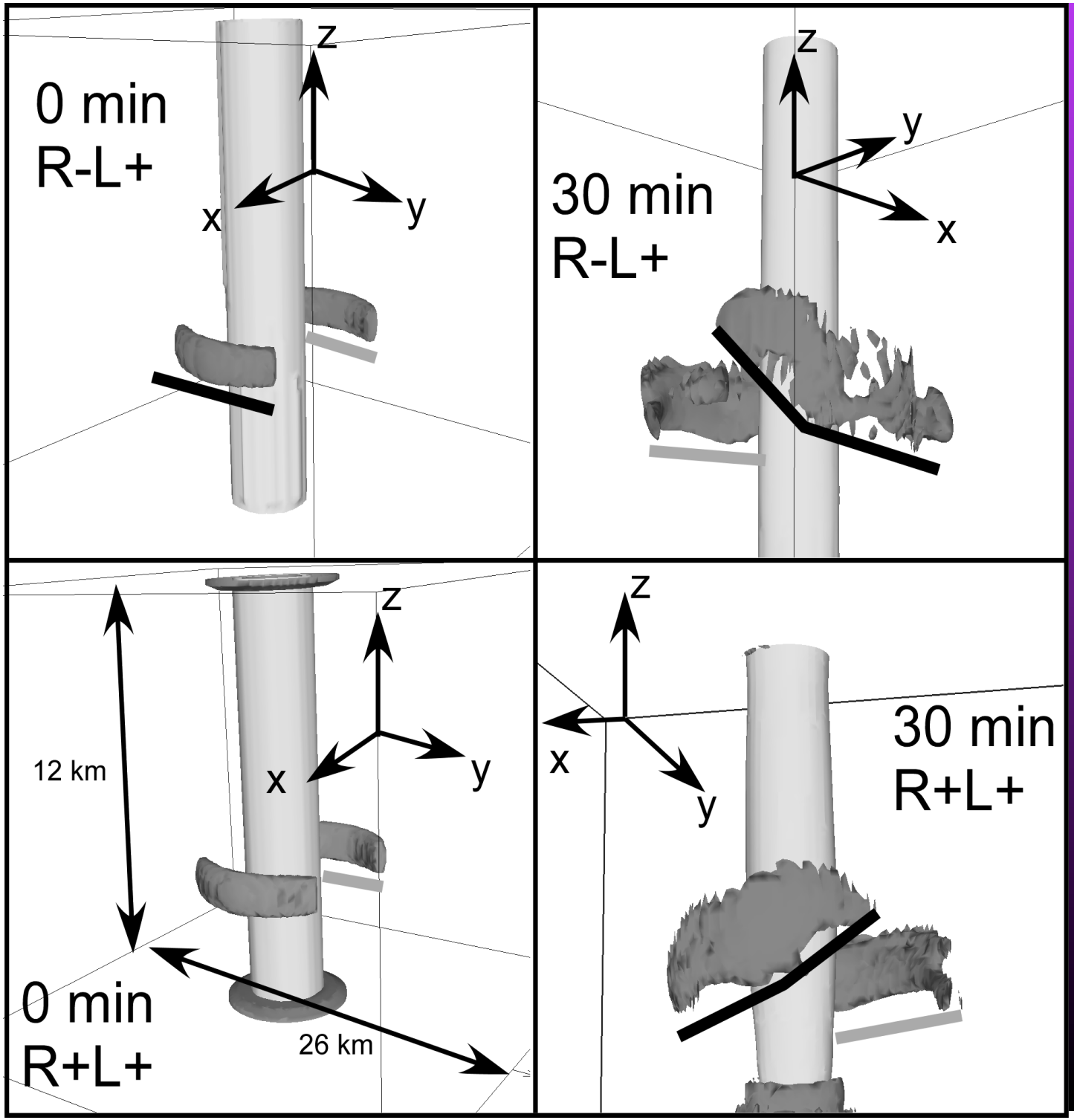
Initial vorticity distributions

Simulation	Rankine vorticity	Curl of horizontal vorticity
R+L+	> 0	> 0
R+L-	> 0	< 0



Horizontal loop slightly offset to test for asymmetry in vortex response

(The more embedded vortex should tilt faster.)



White surface:
Z- vorticity

Dark surface:
X,Y vorticity magnitude

$$QH = \sqrt{(\omega_x^2 + \omega_y^2)}$$

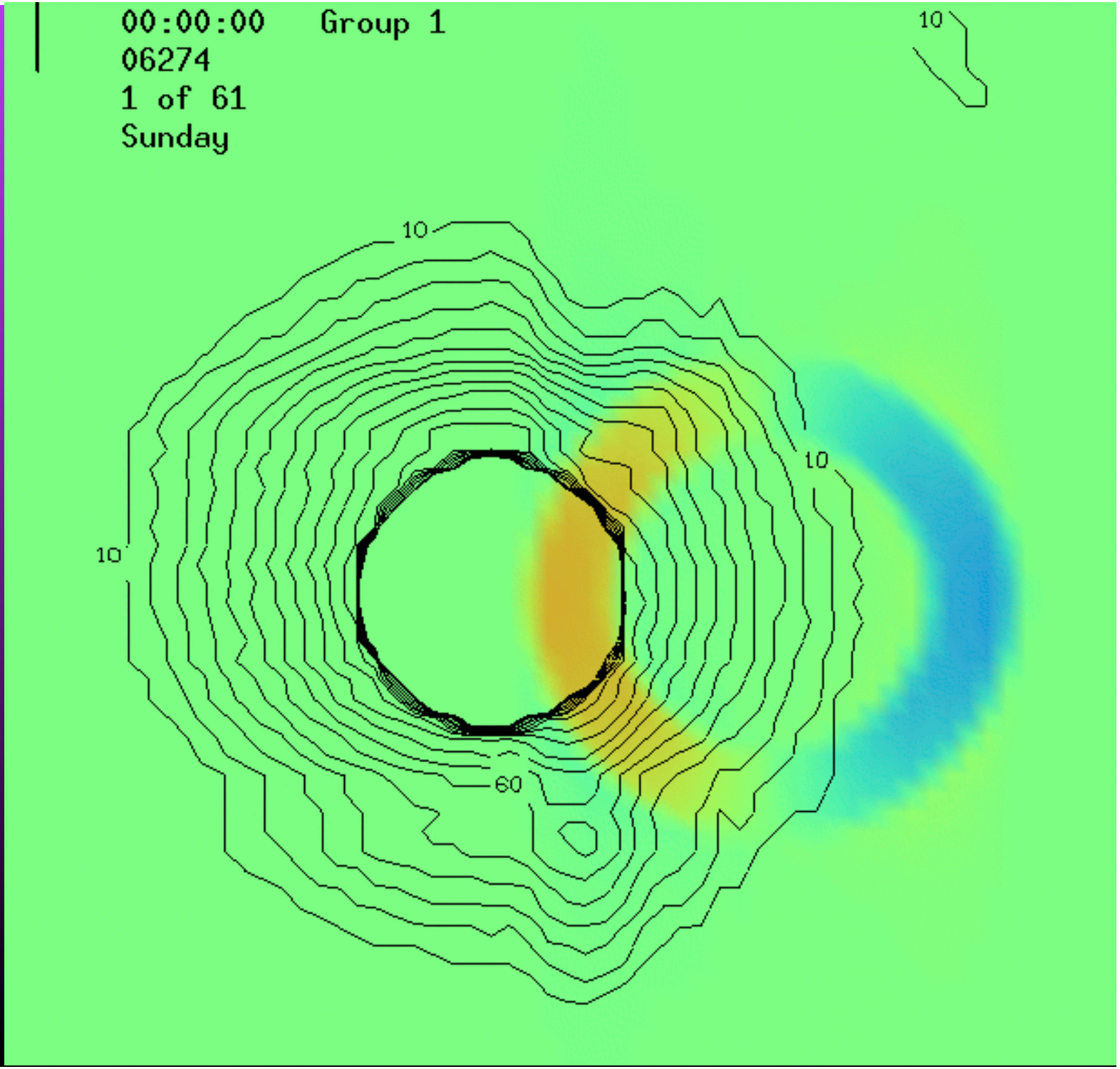
black/grey lines:

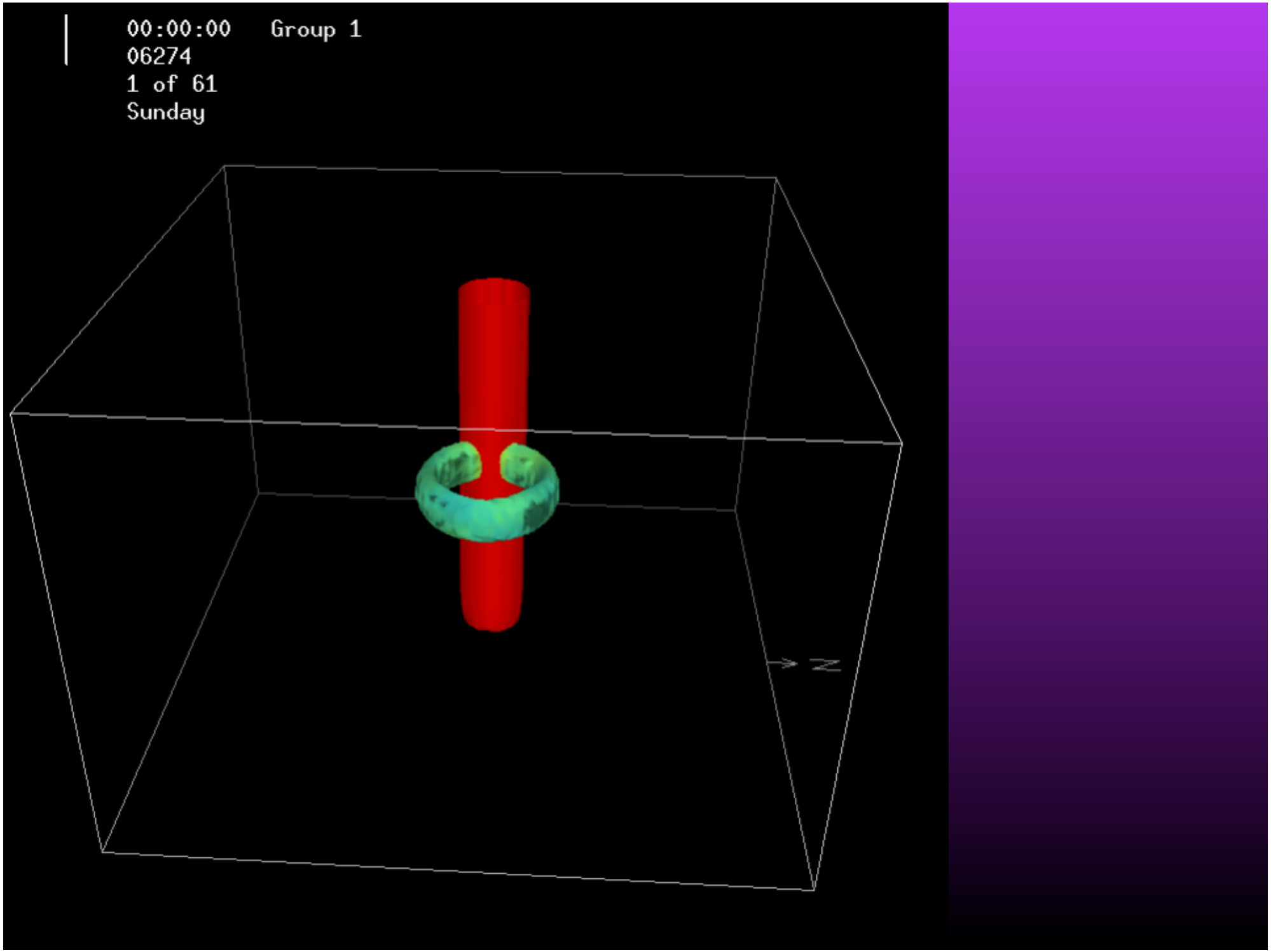
embedded/ non-embedded loop segments

Something perhaps a little more familiar:

R+ L- …

…but L- is embedded like an RFD loop near a meso-cyclone.

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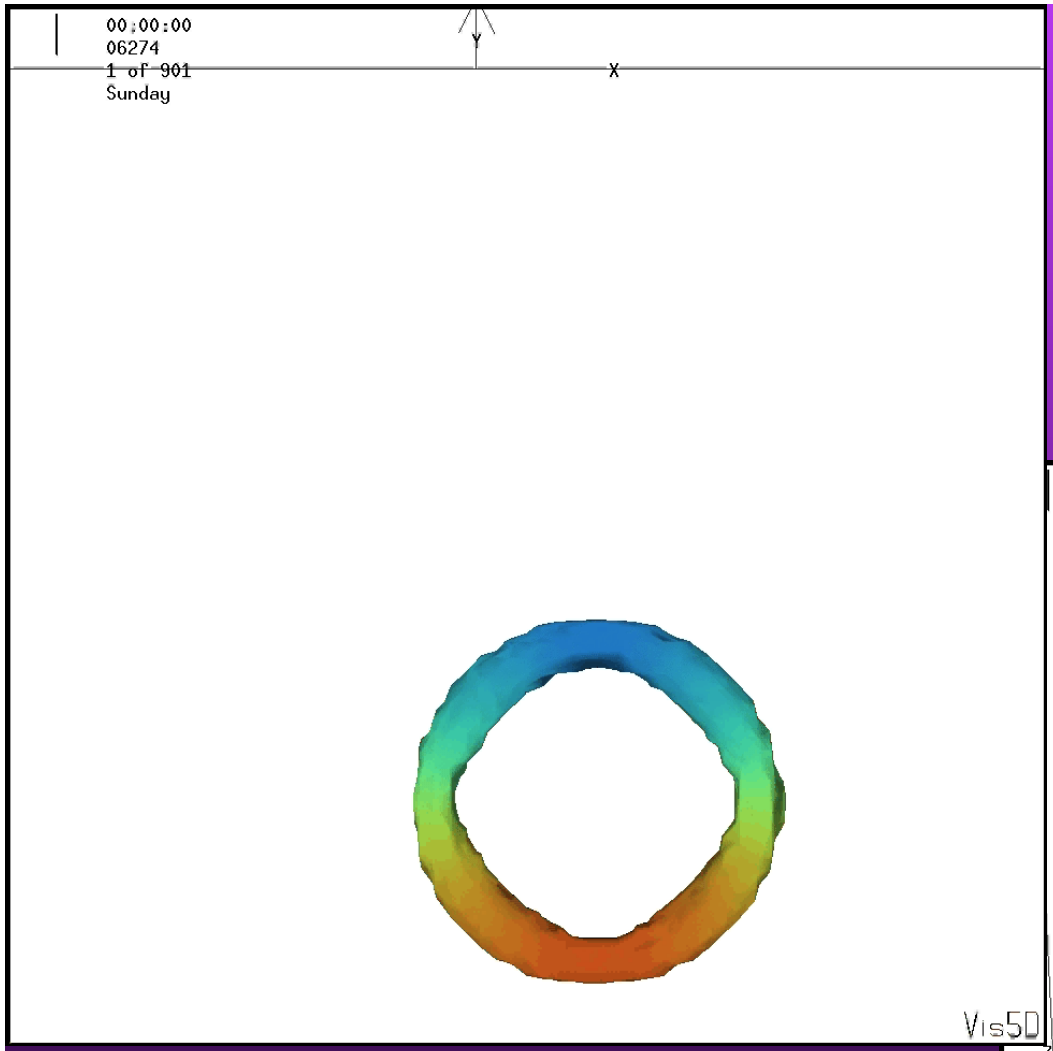
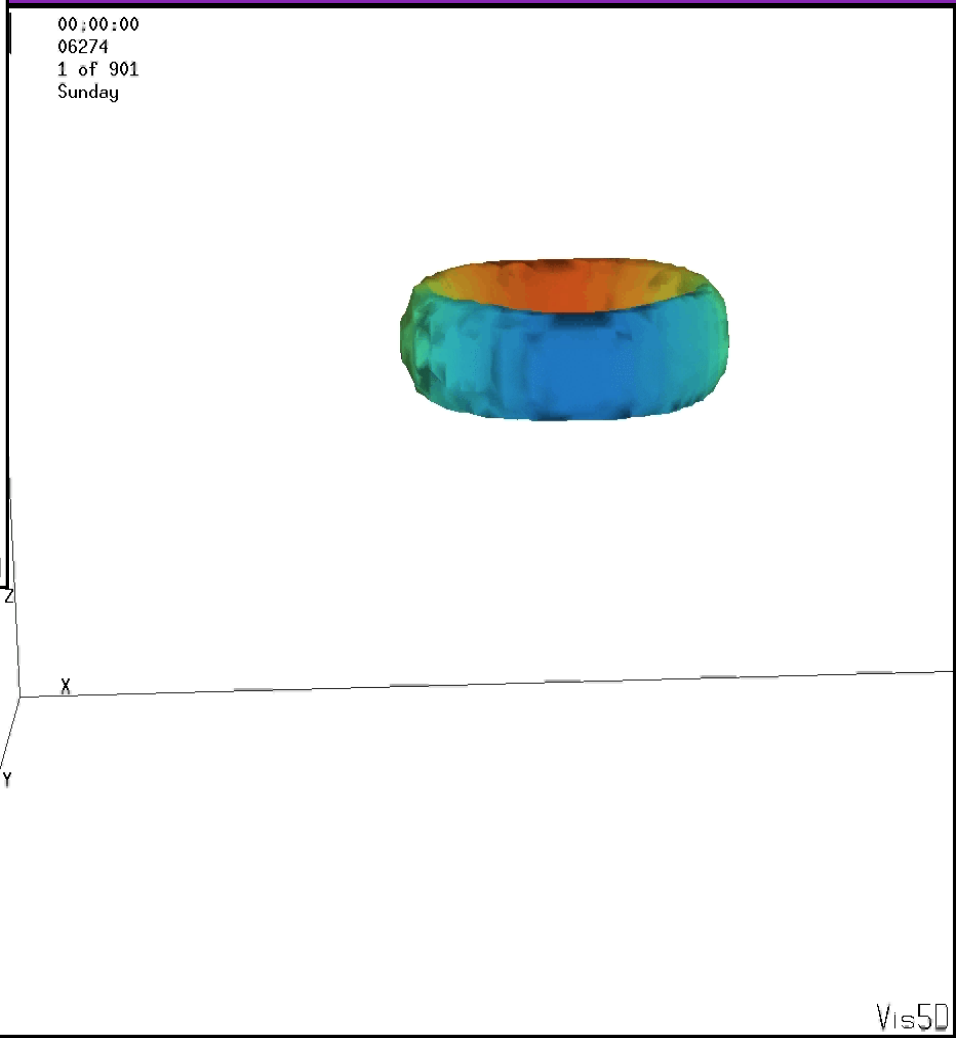
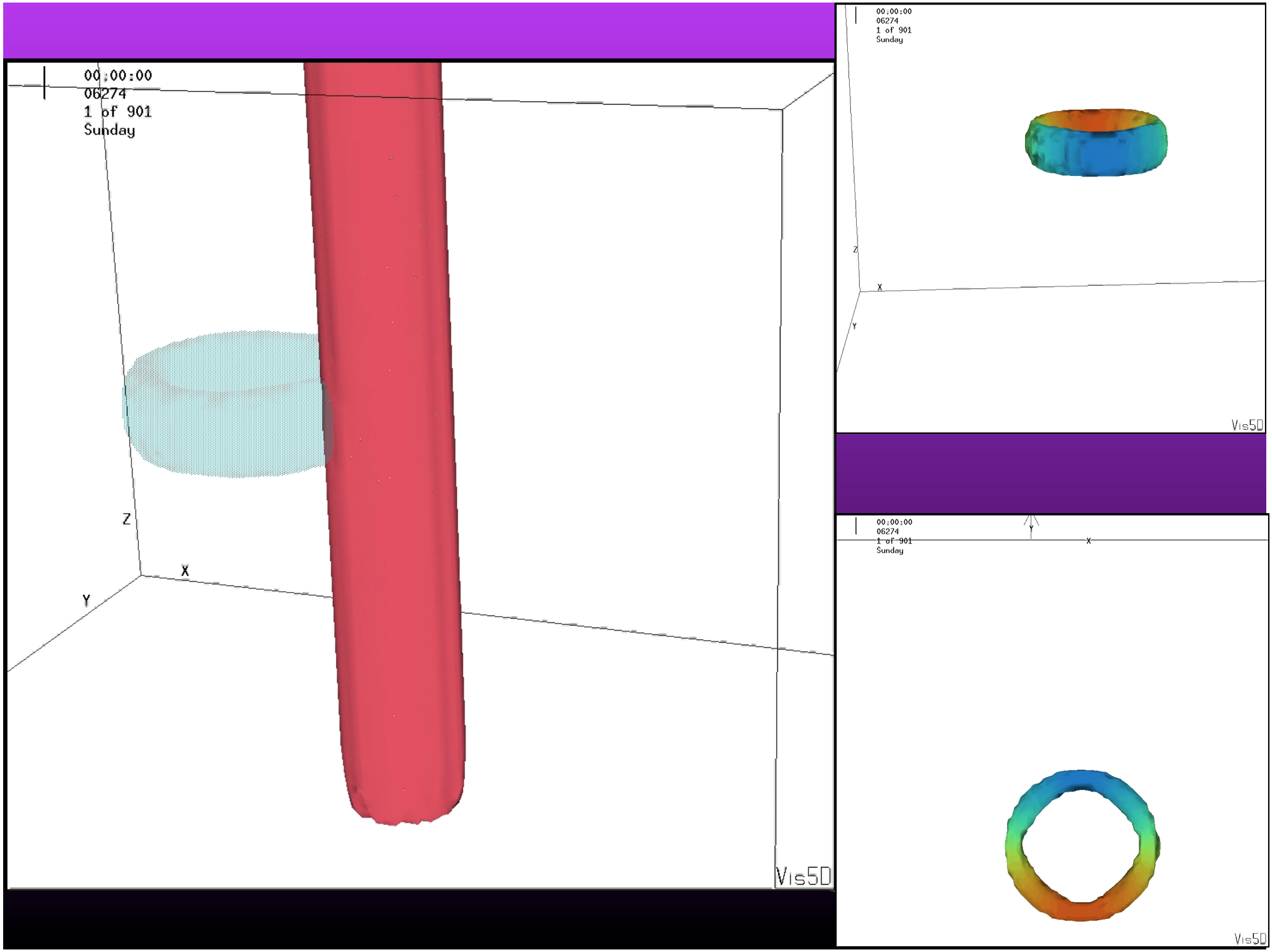


$$QH = \sqrt{\left(\omega_x^2 + \omega_y^2\right)}$$

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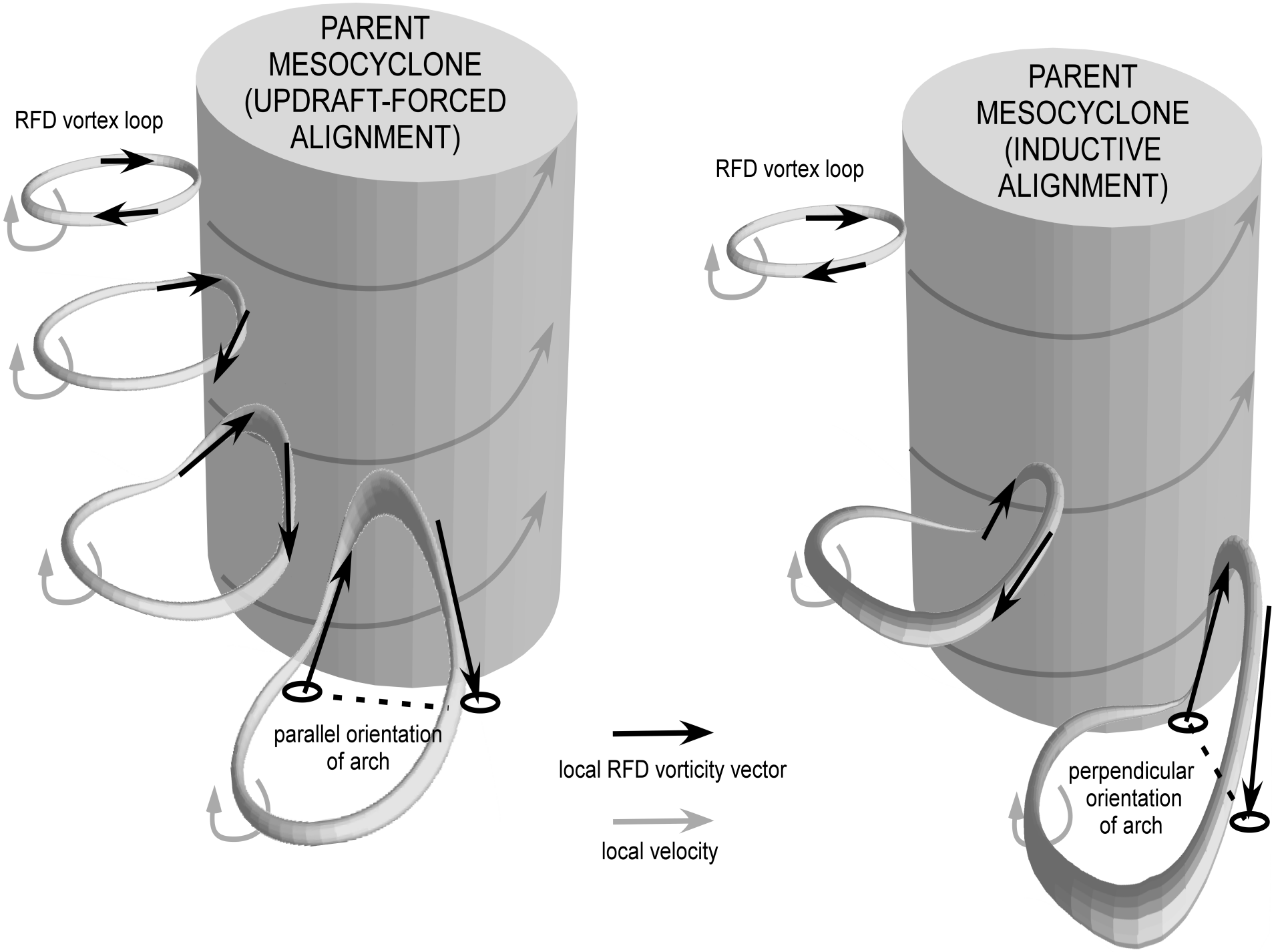
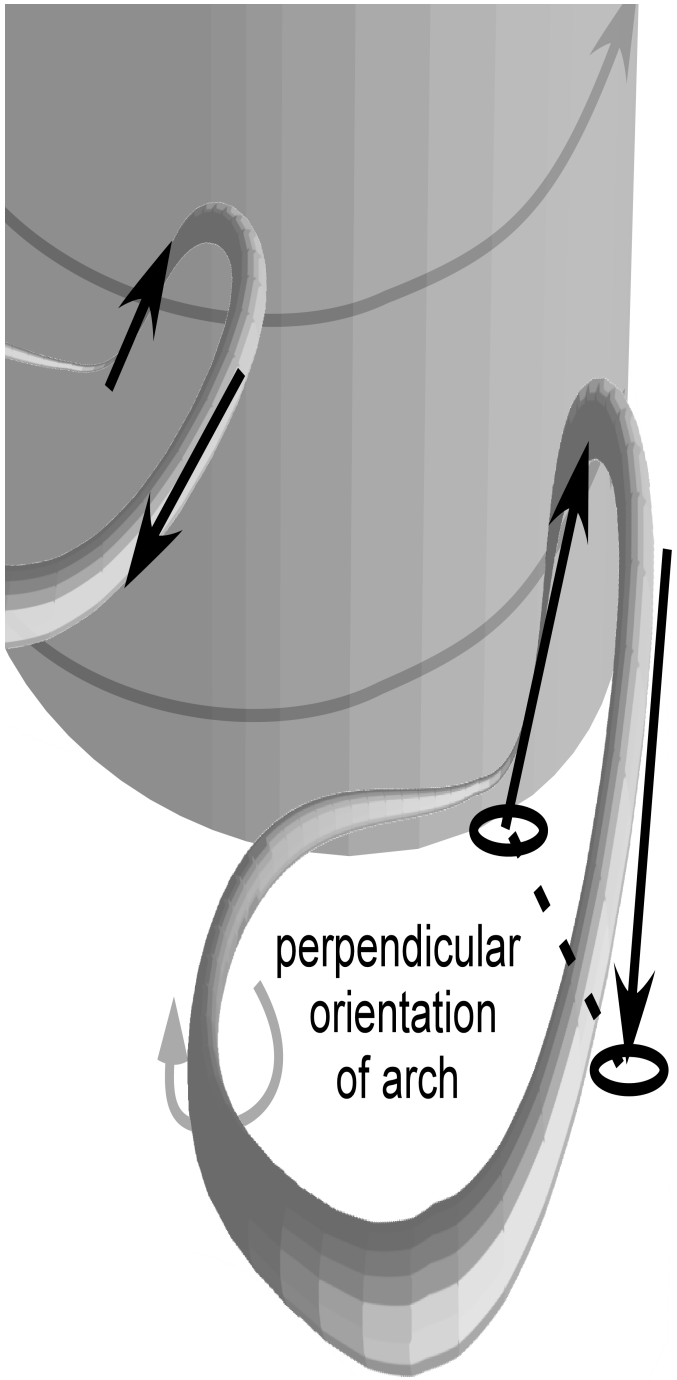

PARENT MESOCYCLONE (UPDRAFT-FORCED ALIGNMENT)
RFD vortex loop
parallel orientation of arch
PARENT MESOCYCLONE (INDUCTIVE ALIGNMENT)
RFD vortex loop
perpendicular orientation of arch
local RFD vorticity vector
local velocity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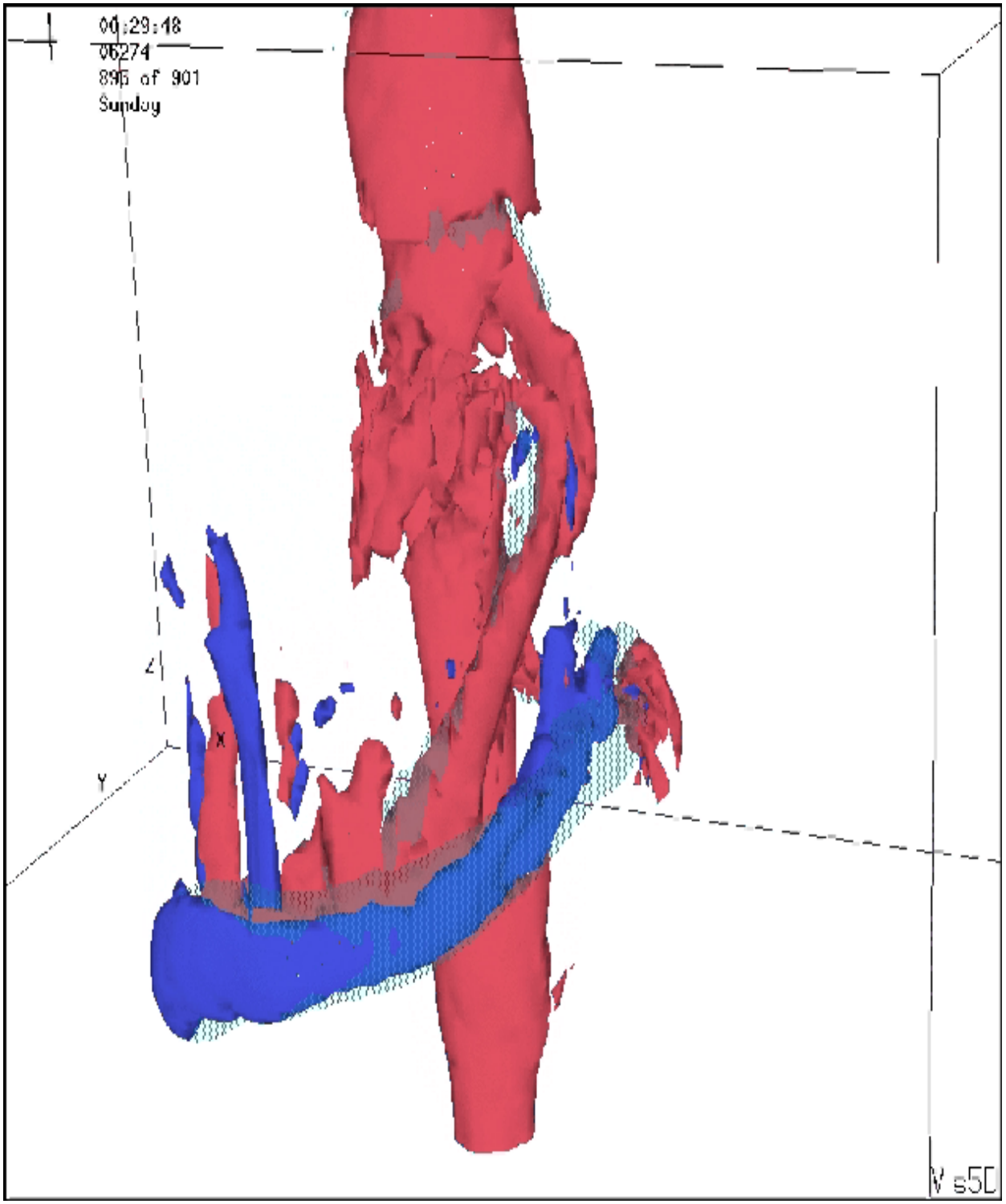
orientation
of arch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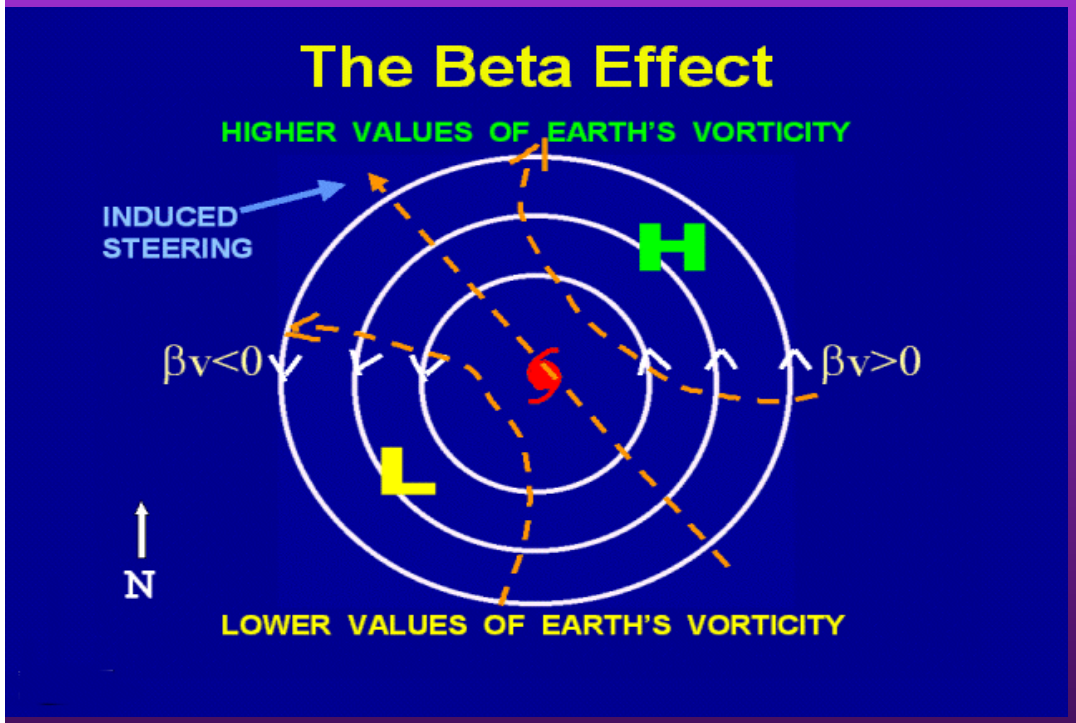
Is there an EM analogy for this?

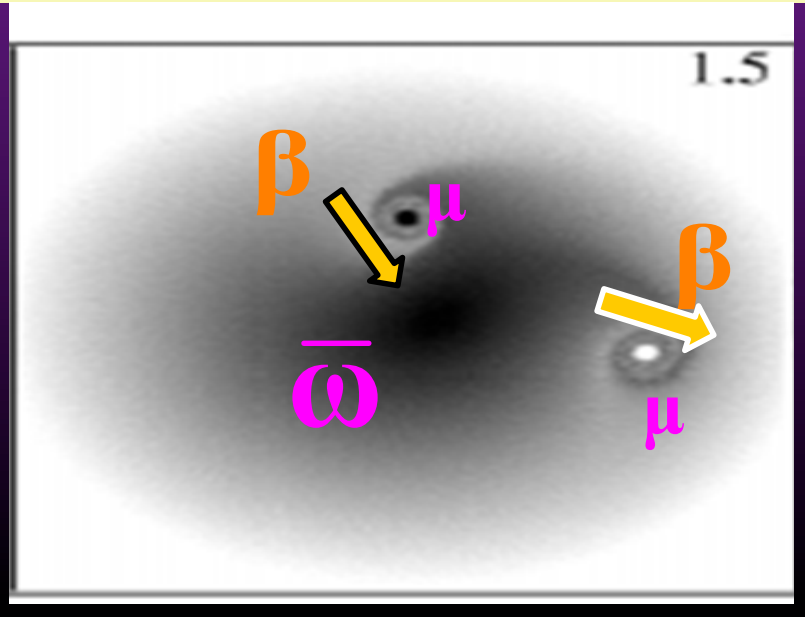


Analogous Variables	
u l	A E
ω	B

$$\mathbf{F}_l = \nabla(\mathbf{m}\cdot\mathbf{B})$$

$$\mathbf{F} = \nabla(\mu\bar{\omega}) \quad \beta$$

- Yes, indeed!



So, what of all this?

When considering scale interaction of vorticity, a simple rule seems to be:

VORTICITY SEEKS ITS OWN.

So, what of all this?

- With insight from the analogy, we are working on a new model of how fundamentally **3-D**, **nonlinear**, inertial/ vortical **scale interaction** occurs in the context of tornadogenesis.

- The effect described here could act to naturally keep a vortex edge (like in a tornado) sharp…a *physically-based* vorticity confinement or ‘backscatter’.

So, what of all this?

Basically, we are moving toward a dynamically (rather than statistically) based closure to the finite-differenced system:

one that physically describes the dynamical linkage (self-similarity) through the inertial scales.

Summary

- While RFD horizontal vorticity can be aligned into the vertical by differential lifting from an updraft, horizontal vorticity embedded within larger rotation (such as the mesocyclone) can naturally align through angular momentum conservation
- This process, among others, is dynamically and mathematically analogous to the electromagnetic version of scale interaction
- These mechanisms may help in developing a new, dynamically-based model of nonlinear inertial (vortex-vortex) scale interactions

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