APPLICATION OF THE SHEAR- AND CURVATURE VORTICTY EQUATIONS TO THE MID-LEVEL MESOCYCLOGENESIS

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I. INTRODUCTION

The interaction of a convective updrafts with ambient vertical wind shear has been thoroughly investigated durig the past decades (e.g., Barnes, 1970; Rotunno and Klemp, 1982; Davies-Jones, 1984; Rotunno and Klemp, 1985, among many others). In these studies, it is shown that the source of the initial mid-level rotation is horizontal shear vorticity associated with the vertical wind shear, which is tilted into the vertical by the convective updraft. This concept is well established and confirmed observationally and numerically. What the description in terms of vorticity does not reveal, however, is how an initially sheared flow is transformed into a coherent vortex. By employing the shear and curvature vorticity equations, it can be shown that in the often investigated archetypal cases of streamwise and crosswise vorticity in the thunderstorm inflow, no vortex develops via the tilting process. Rather, vorticity tilting results in vertical shear vorticity in case of crosswise vorticity in the inflow, and in curvature vorticity in case of a purely helical inflow. Since a vortex requires the presence of both, shear *and* curvature vorticity, part of the shear vorticity must be converted to curvature (crosswise inflow case), and part of the curvature vorticity must be converted to shear (streamwise inflow case). The conversion terms are determined by the pressure field and the velocity field. It is important to note that this analysis is in no respect inconsistent with the perspective of the full vorticity (rather than its shear and curvature components); rather, it provides deeper insight than the mere vorticity perspective. The often-employed picture showing vortex lines being deformed by the updraft, albeit correct, does not reveal these details.

II. SHEAR AND CURVATURE VORTICITY EQUATIONS

The shear and curvature equations have only been applied to synoptic-scale features in the formal literature, to the author's knowledge (Pichler and Steinacker, 1987; Bell and Keyser, 1993). These equations have been expressed in pressure coordinates or isentropic coordinates (Hollmann, 1958; Pichler and Steinacker, 1987; Bell and Keyser, 1993; Bleck, 1991; Viúdez and Haney, 1996). To apply them to a supercell, they have been written in height coordinates. It is sufficient to express them in an inertial coordinate system, as the earth's rotation plays only a negligible role in the initial mid-level mesocyclogenesis. Also, solenoidal generation of vertical vorticity can be neglected. Then, the following equations can be derived (for a detailed derivation, Dahl, 2006, and Viudez and Haney, 1993):

$$\frac{D\zeta_c}{Dt} = -\zeta_c\left(\nabla_h \cdot \vec{v}\right) + \vec{\omega}_{sw} \cdot \nabla_h w - c_p\theta_0 \frac{\partial^2 \pi}{\partial n \partial s} + c_p\theta_0 \frac{1}{V}\frac{\partial V}{\partial s}\frac{\partial \pi}{\partial n} \tag{1}$$

and

$$\frac{D\zeta_s}{Dt} = -\zeta_s\left(\nabla_h \cdot \vec{v}\right) + \vec{\omega}_{cw} \cdot \nabla_h w + c_p\theta_0 \frac{\partial^2 \pi}{\partial n \partial s} - c_p\theta_0 \frac{1}{V}\frac{\partial V}{\partial s}\frac{\partial \pi}{\partial n}. \tag{2}$$

ζ_c and ζ_s are the shear and curvature vorticity, respectively, $\vec{v}$ is the horizontal velocity vector, w is the vertical velocity, V is the magnitude of the horizontal velocity vector; $\vec{\omega}_{sw}$ and $\vec{\omega}_{cw}$ are the horizontal streamwise and crosswise vorticity vectors, respectively. c_p is the specific heat of air at constant pressure, and θ_0 is the potential temperature. π is the dimensionless pressure given by Exner's function. n and s are the directions normal and tangential to the streamlines, respectively.

The first terms on the *rhs* of both equations, (1) and (2), are the divergence terms, which - just as in the full vorticity equation - describe how a convergent or divergent flow field alters the vorticity (either shear or curvature). Note that, e.g., convergence is not able to create curvature vorticity if initially there was merely shear vorticity, and vice versa. The second terms are the tilting terms. Vertical curvature vorticity is created if the horizontal vorticity is purely streamwise. Vertical shear vorticity is created if the horizontal vorticity is purely crosswise. This means, that in these often-discussed cases, no coherent vortex forms. The last two terms only differ in the signs, which identifies them as conversion or interchange terms. They require the pressure field to have just the proper distribution that whenever shear vorticity is depleted, an equal amount of curvature vorticity is generated, and vice versa. Since a coherent vortex always requires both, shear and curvature vorticity, the conversion terms are required if a vortex forms in the updraft after either purely streamwise or purely crosswise vorticity has been tilted into the vertical. A way to visualize what happens during the tilting of shear vorticity, involves isentropic surfaces. The flow is assumed to be unstably stratified and isentropic. The updraft is represented by a hump in the isentropic surfaces, like in Davies-Jones

(1984). Since the flow is isentropic, the parcels remain on their initial isentropic surface. This implies a “flow over an obstacle“ analogy, which is an appropriate model as long as the amplitude of the perturbation is small, i.e., in the early stages of the supercell's life.

III. METHODOLOGY

Based on the above theoretical analysis, a simple conceptual model can be developed of what actually happens when horizontal shear vorticity is tilted into the vertical. It is assumed that the flow is isentropic, while flowing across the perturbed isentropes. Based on these assumptions, the horizontal velicity field can be constructed for stream- and crosswise vorticity cases. This heuristic approach confirms the validity of the above interpretation.

If a vertical vortex is to be established, shear-to-curvature conversions (crosswise vorticity), and curvature-to-shear conversions (streamwise vorticity) need to take place, respectively. These depend on the velocity and pressure fields. The pressure field for a Boussinesq flow is given by (e.g., Davies-Jones, 2002; Appendix of Dahl, 2006)

$$-\frac{1}{\rho}\nabla^2 p' = \left|\vec{\vec{D}}\right|^2 - \frac{1}{2}\left|\vec{\omega}\right|^2 - \frac{\partial B}{\partial z},$$

where p' is the perturbation pressure, ρ is the density, $\vec{\vec{D}}$ is the rate of strain tensor, $\vec{\omega}$ is the vorticity vector, and B is the buoyancy.

While the above conceptual model provides some insight, more advanced analysis techniques are required to investigate the conversion terms. A numerical analysis is the preferred path, providing the pressure and velocity fields at every time step, which can be used to calculate the conversion terms. Interestingly, the conversion terms act in opposite directions in the streamwise- and crosswise vorticity cases.

IV. SUMMARY AND FUTURE RESEARCH

It has been demonstrated that tilting of horizontal shear vorticity does, in general, not produce a vertical vortex. Rather, vertical shear vorticity is produced if the thunderstorm inflow possesses crosswise vorticity, and vertical curvature vorticity is produced if the thunderstorm inflow carries streamwise vorticity. In order to complete a vortex, substantial shear-curvature vorticity conversion are required. These depend on the pressure and velocity fields. Further research is planned to demonstrate the role of the conversion terms by employing a numerical model like the WRF. Also, quantities other than vorticity may be used to investigate storm rotation. Vorticity is only a local measure of rotation, and thus in principle inadequate to identify macroscopic vortices (wave and shearing motions also possess vorticity); see also Rotunno and Klemp (1985). Recently, a 2D "fluid trapping" formalism has been developed by Cohen and Shultz (2005): Two fluid parcels, which initially may become more and more separated from one another with time, may become trapped in certain flow regimes. This is what also happens to air parcels in the inflow of a supercell thunderstorm: A vertically sheared flow is clearly associated with an increase of the magnitude of the separation vector of two initially neighboring parcels. When encountering an updraft, the parcels are, at least temporarily, trapped in the updraft while being part of the mesocyclone. However, this formalism needs to be extended to three dimensions if it is to be applied to supercells, which has not been attempted yet, to the author's knowledge.

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